MATH 327, HOMEWORK #6

Problem 1. Find a closed form solution for the sequence defined recursively by

$$a_0 = 7$$

and, for $n \geq 1$,

$$a_n = \frac{1}{2}a_{n-1} + 3.$$

Problem 2. How many natural numbers between 1 and 1000 are divisible by at least one of 6, 10 and 15.

Problem 3.

(a) List all the sequences of zeros and ones, of length 2, that do not have both first and last element a one.
(b) List all the sequences of zeros and ones, of length 4, that do not have both first and last element a one.
(c) Suppose $n \geq 2$. How many sequences of zeros and ones, of length n, are there that do not have both first and last element a one.

Problem 4.

(a) List all the sequences in 1 and 2, of length 3, that remain equal to themselves when reversed (Palindromes).
(b) List all the sequences in 1, 2 and 3, of length 4, that remain equal to themselves when reversed.
(c) How many sequences in $1, 2, \ldots, m$, of length n, are there that remain equal to themselves when reversed.

Problem 5. Let $A = \{1, 2, \ldots, 8\}$ and $B = \{1, 2, \ldots, 2000\}$. Find the number of functions that there are from A to B that have range that either is a one-element set or that does not contain the element 1.

Problem 6. Consider a rectangle of length 4 and width 1.

(a) Show that given any 5 points in this rectangle, there must be at least one pair that are of distance at most $\sqrt{2}$ from each other.
(b) Show that given any 9 points in this rectangle, there must be at least one pair that are of distance at most $\frac{\sqrt{5}}{2}$ from each other.