MATH 311 - Vector Analysis
Homework # 31

Problems in Schaum's Book: (p. 133-134) 61*, 71*, 72*

61. Show that Green's second identity can be written

$$\iiint_V \left(\phi\nabla^2\psi - \psi\nabla^2\phi\right)\, dV = \iint_{\mathcal{S}} \left(\phi\frac{d\psi}{dn} - \psi\frac{d\phi}{dn}\right)\, dS.$$

71. Prove

$$\iiint_V \nabla\phi\cdot\vec{A}\, dV = \iint_{\mathcal{S}} \phi\vec{A}\cdot\vec{n}\, dS - \iiint_V \phi\nabla\cdot\vec{A}\, dV.$$

72. Let $\vec{r}$ be the position vector of any point relative to an origin $\vec{O}$. Suppose ϕ has continuous derivatives of order two, at least, and let $\mathcal{S}$ be a closed surface bounding a volume V. Denote ϕ at $\vec{O}$ by ϕ_0. Show that

$$\iint_{\mathcal{S}} \left[\frac{1}{r}\nabla\phi - \phi\nabla\left(\frac{1}{r}\right)\right]\cdot d\vec{S} = \iiint_V \frac{\nabla^2\phi}{r}\, dV + \alpha,$$

where $\alpha = 0$ or $4\pi\phi_0$ according as $\vec{O}$ is outside or inside V. Remember $r = |\vec{r}|$.