MATH-163, Section 001, Spring 1999
Review for Midterm # 3

1. Evaluate the following limits:

(a) $\lim_{n\to\infty} \dfrac{2n+1}{1-3n}$

(b) $\lim_{n\to\infty} \cos(1/n)$

(c) $\lim_{n\to\infty} \dfrac{n}{e^n}$

2. Decide if the following series converge or diverge. If they converge, do they converge absolutely? Find the sum, if possible.

(a) $\sum_{n=0}^{\infty} \dfrac{5^n}{7^{n+1}}$

(b) $\sum_{n=1}^{\infty} \cos(1/n)$

(c) $\sum_{n=1}^{\infty} \dfrac{(-1)^n}{\sqrt{n}}$

3. Given the following power series find the radius of convergence and the interval of convergence.

(a) $\sum_{n=0}^{\infty} \dfrac{x^n}{n+1}$

(b) $\sum_{n=0}^{\infty} \dfrac{(x-2)^n}{3^n}$

4. Find the Taylor series at $a = 2$ for the given function. (write out at least the first four terms!).

$$f(x) = \sin(2x-4), \qquad a = 2$$

5. It is known that for $|x| < \infty$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + \cdots$$

(a) Find $P_4(x)$ the Taylor polynomial of order 4 of $f(x) = e^x$ at $a = 0$. Use Taylors formula to find the corresponding remainder $R_4(x)$.

(b) Evaluating at $x = -1$ we get:

$$e^{-1} = \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!} + \cdots$$

How many terms do we have to add up so that the partial sum is accurate to within an error of 10^{-2} ?

6. **(a)** Show that the following series converges

$$\sum_{n=1}^{\infty} \frac{n!\, n^n}{(2n)!}$$

(b) Find the Taylor series at $a = 0$ of

$$f(x) = 1 + 3x - x^2 + x\cos x$$

(c) It is known that

$$\frac{1}{1+x^2} = 1 - x^2 + x^4 - x^6 + \cdots \qquad |x| < 1$$

Find the Maclaurin series of $\arctan x$. Indicate the interval of convergence. (Hint: remember that $\frac{d}{dx}\arctan x = \frac{1}{1+x^2}$.)

MIDTERM # 3: Wednesday April 14th at 8:00am - Sections 8.1-8.10.