MATH 163-001 - CALCULUS II - Spring 1999

Review Problems #1

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MIDTERM EXAM # 1: Wednesday Feb 17th, 1999

1. Given the function $f(x) = x^2 + 1, \quad x \geq 0$.

(a) Find the inverse, f^{-1}, expressed as a function of x. Indicate what are the domain and range of f^{-1}.
(b) Find the value of $\frac{d}{dx} f^{-1}$ at the point $x = 2$.
(c) Draw the graphs of f and f^{-1} together.

2. Suppose $g = f^{-1}$ and f have derivatives everywhere. If $f(2) = 3$, $f'(2) = 5$ and $f'(3) = 4$, what is $g'(3)$?

3. Solve for x: $\displaystyle\int_2^{15} \frac{1}{t} dt + \int_5^6 \frac{1}{t} dt = 2 \int_1^x \frac{1}{t} dt.$

4. Find the derivatives with respect to x of the following functions:

(a) $f(x) = \sin^{-1}(\cos x)$ (can you explain your answer?)

(b) $g(x) = x^{x+1}$ **(c)** $h(x) = \tan^{-1}(x^2)$

(d) $f(x) = e^x (x+1)^4 \sqrt{(3x-2)}$ (use logarithmic differentiation).

5. Evaluate the following integrals (show your work!):

(a) $\displaystyle\int_0^{\pi/2} \frac{\cos t}{2 + \sin t} dt$ **(b)** $\displaystyle\int_0^1 x e^{x^2} dx$

(c) $\displaystyle\int_0^2 \frac{dx}{\sqrt{2x - x^2}}$ **(d)** $\displaystyle\int \frac{e^{2x}}{e^x + e^{3x}} dx$

6. Find the following limits:

(a) $\displaystyle\lim_{x \to 0} \frac{\sin^{-1} x}{x}$ **(b)** $\displaystyle\lim_{x \to \infty} \left(1 - \frac{1}{x}\right)^x$

7. Suppose you deposit \$700 in a bank account that pays interest compounded continuously. Suppose that after 5 years you have \$1050.

(a) What was the rate of interest at which the money was compounded?
(b) How long will it take for your money to triple?

8. Order the following funtions from slowest growing to fastest growing at infinity: x, $\cosh x$, $\ln(x^3)$, $x \tan^{-1} x$. Are any of them growing at the same rate?

9. Solve for x (in terms of t if necessary):

(a) $\tan(\cos^{-1} x) = \sqrt{3}$; **(b)** $\ln((x+1)e^2) = 3t$

10. Solve the initial value problem:

$$\frac{dy}{dx} = 4e^x \sinh x, \qquad y(0) = 2.$$