

<b>Table: <math>\mathbb{Q}</math>-Homology 7-Spheres <math>M_{\mathbf{w},d}^7</math> admitting S-E Structures</b>			
$\mathbf{w} = (w_0, w_1, w_2, w_3, w_4)$	$d$	$\mu$	Order of $H_3(M_{\mathbf{w},d}^7, \mathbb{Z})$
(17, 34, 75, 125, 175)	425	4416	$582622237229761 = 17^{12}$
(17, 238, 381, 635, 889)	2159	16272	$118587876497 = 17^9$
(19, 57, 100, 125, 175)	475	3168	$16983563041 = 19^8$
(49, 334, 525, 668, 763)	2338	4995	$37259704 = 2^3 \cdot 167^3$
(49, 573, 1862, 2483, 4393)	9359	36720	$282475249 = 7^{10}$
(50, 65, 73, 73, 105)	365	1152	$28398241 = 73^4$
(52, 127, 381, 533, 559)	1651	5040	$260144641 = 127^4$
(55, 160, 373, 373, 905)	1865	5932	$19356878641 = 373^4$
(87, 558, 687, 1331, 1331)	3993	4320	$1771561 = 11^6$
(93, 459, 780, 1331, 1331)	3993	4320	$1771561 = 11^6$
(93, 1011, 2298, 3401, 3401)	10203	13600	$11566801 = 19^2 \cdot 179^2$
(97, 1531, 2201, 2775, 3253)	9856	9855	$9856 = 2^7 \cdot 7 \cdot 11$
(99, 318, 465, 881, 881)	2343	3520	$776161 = 881^2$
(101, 439, 559, 579, 619)	2296	2295	$2296 = 2^3 \cdot 7 \cdot 41$
(101, 1597, 1996, 2695, 3693)	10081	10080	$10081 = 17 \cdot 593$
(101, 1597, 2096, 2495, 3793)	10081	10080	$10081 = 17 \cdot 593$
(101, 1697, 2296, 2695, 4093)	10881	10880	$10881 = 3^3 \cdot 13 \cdot 31$
(103, 1321, 2337, 2845, 3251)	9856	9855	$9856 = 2^7 \cdot 7 \cdot 11$
(108, 267, 507, 881, 881)	2643	3520	$776161 = 881^2$
(109, 1616, 2047, 2693, 4417)	10881	10880	$10881 = 3^3 \cdot 13 \cdot 31$
(111, 329, 407, 423, 470)	1739	1728	$1369 = 37^2$
(111, 658, 2303, 3071, 6031)	12173	24600	$35611289 = 7^3 \cdot 47^3$
(111, 768, 2523, 3401, 3401)	10203	13600	$11566801 = 19^2 \cdot 179^2$
(113, 1115, 6021, 8362, 9589)	25199	50064	$1142897 = 113^3$
(113, 1561, 3345, 8362, 11819)	25199	50064	$1142897 = 113^3$
(115, 341, 523, 591, 727)	2296	2295	$2296 = 2^3 \cdot 7 \cdot 41$
(115, 797, 949, 987, 2050)	4897	4896	$4897 = 59 \cdot 83$
(125, 1732, 4577, 5567, 6433)	18433	18432	18433
(125, 2599, 4208, 5569, 9901)	22401	22400	$22401 = 3^2 \cdot 19 \cdot 131$
(127, 1888, 2643, 4657, 6671)	15985	15984	$15985 = 5 \cdot 23 \cdot 139$
(127, 2266, 3651, 6043, 8435)	20521	20520	20521
(127, 2392, 3399, 6043, 8561)	20521	20520	20521
(127, 2770, 4407, 7429, 10325)	25057	25056	25057
(129, 511, 1192, 1235, 1831)	4897	4896	$4897 = 59 \cdot 83$
(133, 346, 379, 527, 857)	2241	2240	$2241 = 3^3 \cdot 83$
(136, 337, 421, 455, 893)	2241	2240	$2241 = 3^3 \cdot 83$
(136, 2023, 3237, 5395, 7553)	18343	17280	$289 = 17^2$
(137, 1223, 1427, 2786, 4349)	9921	9920	$9921 = 3 \cdot 3307$
(137, 1495, 1699, 3466, 5301)	12097	12096	12097
(138, 171, 393, 701, 701)	2103	2800	$149401 = 701^2$
(139, 2343, 3721, 6202, 10061)	22465	22464	$22465 = 5 \cdot 4493$

Q-Homology 7-Spheres $M_{\mathbf{w},d}^7$ admitting S-E Structures (cont.)			
$\mathbf{w} = (w_0, w_1, w_2, w_3, w_4)$	$d$	$\mu$	Order of $H_3(M_{\mathbf{w},d}^7, \mathbb{Z})$
(139, 3171, 5101, 8548, 13787)	30745	30744	$30745 = 5 \cdot 11 \cdot 13 \cdot 43$
(141, 652, 1909, 2375, 2701)	7777	7776	$7777 = 7 \cdot 11 \cdot 101$
(141, 1259, 1492, 3031, 4663)	10585	10584	$10585 = 5 \cdot 29 \cdot 73$
(141, 2224, 3475, 4031, 9729)	19599	19600	$19881 = 3^2 \cdot 47^2$
(141, 2363, 3058, 4309, 9729)	19599	19600	$19881 = 3^2 \cdot 47^2$
(143, 194, 209, 231, 291)	1067	1152	$9409 = 97^2$
(143, 1135, 2057, 3476, 5675)	12485	13608	$1288225 = 5^2 \cdot 227^2$
(143, 2981, 6530, 9795, 16467)	35915	39168	$10660225 = 5^2 \cdot 363^2$
(145, 2157, 3451, 5752, 9347)	20851	20160	$841 = 29^2$
(146, 869, 1955, 2969, 3983)	9921	9920	$9921 = 3 \cdot 3307$
(147, 207, 230, 245, 299)	1127	1152	$2401 = 7^4$
(147, 255, 1056, 1457, 1457)	4371	5824	$2122849 = 31^2 \cdot 47^2$
(154, 535, 739, 1427, 2115)	4969	4968	4969
(154, 763, 1297, 2975, 3891)	9079	10368	$1682209 = 1297^2$
(155, 921, 1612, 2149, 4681)	9517	9792	$94249 = 307^2$
(155, 1075, 3532, 5835, 7064)	17660	21186	$12475024 = 3532^2$
(155, 2309, 7543, 10006, 12469)	32481	32480	$32481 = 3^4 \cdot 401$
(155, 2617, 8467, 11392, 14163)	36793	36792	36793
(157, 269, 637, 665, 1090)	2817	2816	$2817 = 3^2 \cdot 313$
(157, 436, 1401, 1775, 1993)	5761	5760	$5761 = 7 \cdot 823$
(157, 545, 1051, 1401, 2608)	5761	5760	$5761 = 7 \cdot 823$
(157, 883, 1558, 2597, 4311)	9505	9504	$9505 = 5 \cdot 1901$
(157, 1195, 2182, 3689, 6027)	13249	13248	13249
(157, 2339, 5146, 7641, 12943)	28225	28224	$28225 = 5^2 \cdot 1129$
(157, 2651, 5770, 8733, 14659)	31969	31968	$31969 = 7 \cdot 4567$
(159, 2365, 3784, 6307, 12455)	25069	25488	$223729 = 11^2 \cdot 43^2$
(163, 1939, 2747, 6786, 8887)	20521	20520	20521
(163, 1939, 3070, 5171, 10179)	20521	20520	20521
(163, 2101, 3394, 5657, 11151)	22465	22464	$22465 = 5 \cdot 4493$
(166, 1237, 4371, 5773, 7175)	18721	18720	$18721 = 97 \cdot 193$
(166, 1399, 2551, 5513, 7077)	16705	16704	$16705 = 5 \cdot 13 \cdot 257$
(167, 717, 1324, 1765, 3805)	7777	7776	$7777 = 7 \cdot 11 \cdot 101$
(169, 2015, 6549, 8732, 10915)	28379	26208	$169 = 13^2$
(170, 1267, 3041, 4477, 7687)	16641	16640	$16641 = 3^2 \cdot 43^2$
(171, 247, 556, 695, 973)	2641	2520	$361 = 19^2$
(175, 271, 299, 306, 751)	1801	1800	1801
(175, 289, 925, 2312, 3525)	7225	14688	$180124137569 = 17^6$
(175, 2434, 10605, 13387, 15995)	42595	43776	$1481089 = 1217^2$
(176, 295, 317, 481, 973)	2241	2240	$2241 = 3^3 \cdot 83$
(176, 1135, 1397, 4103, 5675)	12485	13608	$1288225 = 5^2 \cdot 227^2$
(177, 997, 3695, 5044, 6217)	16129	16128	$16129 = 3^6$

Q-Homology 7-Spheres $M_{\mathbf{w},d}^7$ admitting S-E Structures (cont.)			
$\mathbf{w} = (w_0, w_1, w_2, w_3, w_4)$	$d$	$\mu$	Order of $H_3(M_{\mathbf{w},d}^7, \mathbb{Z})$
(177, 2275, 5950, 7175, 15399)	30975	30976	$31329 = 3^2 \cdot 59^2$
(178, 883, 2031, 3973, 5033)	12097	12096	12097
(181, 1613, 4301, 5197, 11110)	22401	22400	$22401 = 3^2 \cdot 19 \cdot 131$
(181, 1618, 7011, 8809, 10607)	28225	28224	$28225 = 5^2 \cdot 1129$
(181, 2338, 10251, 12949, 15467)	41185	41184	$41185 = 5 \cdot 8237 \cdot 131$
(183, 787, 1453, 2422, 4661)	9505	9504	$9505 = 5 \cdot 1901$
(183, 1031, 2608, 4003, 6793)	14617	14616	$14617 = 47 \cdot 311$
(185, 477, 991, 1321, 2788)	5761	5760	$5761 = 7 \cdot 823$
(187, 409, 539, 1320, 2045)	4499	4896	$167281 = 409^2$
(187, 631, 1299, 2746, 3563)	8425	8424	$8425 = 5^2 \cdot 337$
(187, 781, 2306, 3459, 5951)	12683	13824	$1329409 = 1153^2$
(187, 1853, 2594, 6485, 10931)	22049	23328	$1682209 = 1297^2$
(187, 2416, 8177, 10965, 19328)	41072	43470	$5837056 = 2^8 \cdot 151^2$
(191, 235, 433, 509, 904)	2241	2240	$2241 = 3^3 \cdot 83$
(191, 2467, 7990, 10627, 18787)	40041	40040	$40041 = 3^3 \cdot 1483$
(191, 3607, 11770, 15757, 27717)	59041	59040	$59041 = 17 \cdot 23 \cdot 151$
(193, 3247, 4202, 10877, 18335)	36863	36864	$37249 = 193^2$
(194, 539, 1155, 2425, 3157)	7469	7488	$9409 = 97^2$
(194, 693, 847, 2425, 3311)	7469	7488	$9409 = 97^2$
(195, 484, 613, 1291, 2387)	4969	4968	4969
(196, 681, 827, 2383, 3259)	7345	7344	$7345 = 5 \cdot 13 \cdot 113$
(196, 1119, 1411, 4135, 5741)	12601	12600	12601
(196, 2337, 7595, 10127, 17917)	38171	37440	$2401 = 7^4$
(197, 881, 4111, 5188, 6265)	16641	16640	$16641 = 3^2 \cdot 43^2$
(197, 1273, 6071, 7736, 9205)	24481	24480	24481
(199, 305, 377, 628, 1309)	2817	2816	$2817 = 3^2 \cdot 313$
(199, 376, 831, 1603, 2177)	5185	5184	$5185 = 5 \cdot 17 \cdot 61$
(199, 673, 2811, 3880, 4751)	12313	12312	$12313 = 7 \cdot 1759$
(199, 1973, 3157, 7300, 12429)	25057	25056	25057
(203, 1409, 1912, 4931, 8251)	16705	16704	$16705 = 5 \cdot 13 \cdot 257$
(205, 389, 1093, 1389, 2686)	5761	5760	$5761 = 7 \cdot 823$
(205, 1629, 7127, 10588, 12421)	31969	31968	$31969 = 7 \cdot 4567$
(206, 247, 259, 319, 771)	1801	1800	1801
(206, 1331, 4607, 6143, 10955)	23241	23240	$23241 = 3 \cdot 61 \cdot 127$
(207, 2468, 8021, 10695, 21183)	42573	43120	$380689 = 617^2$
(208, 439, 1007, 2091, 2737)	6481	6480	6481
(209, 707, 2038, 3161, 5407)	11521	11520	$11521 = 41 \cdot 281$
(209, 1351, 5092, 6859, 12159)	25669	27000	$1825201 = 7^2 \cdot 193^2$
(211, 339, 436, 985, 1631)	3601	3600	$3601 = 13 \cdot 277$
(211, 1886, 6077, 8173, 16135)	32481	32480	$32481 = 3^4 \cdot 401$
(211, 2306, 7547, 10063, 19915)	40041	40040	$40041 = 3^3 \cdot 1483$

Q-Homology 7-Spheres $M_{\mathbf{w},d}^7$ admitting S-E Structures (cont.)			
$\mathbf{w} = (w_0, w_1, w_2, w_3, w_4)$	$d$	$\mu$	Order of $H_3(M_{\mathbf{w},d}^7, \mathbb{Z})$
(214, 489, 659, 1849, 2551)	5761	5760	$5761 = 7 \cdot 823$
(217, 731, 1075, 2752, 4557)	9331	9504	$47089 = 7^2 \cdot 31^2$
(217, 817, 946, 2795, 4557)	9331	9504	$47089 = 7^2 \cdot 31^2$
(217, 2795, 7310, 13115, 23219)	46655	46656	$47089 = 7^2 \cdot 31^2$
(217, 3440, 5375, 14405, 23219)	46655	46656	$47089 = 7^2 \cdot 31^2$
(220, 237, 1021, 1367, 1477)	4321	4320	$4321 = 29 \cdot 149$
(221, 317, 805, 1073, 2194)	4609	4608	$4609 = 11 \cdot 419$
(221, 2416, 5491, 13617, 19328)	41072	43470	$5837056 = 2^8 \cdot 151^2$
(223, 256, 563, 1041, 1519)	3601	3600	$3601 = 13 \cdot 277$
(223, 2212, 3539, 9511, 15261)	30745	30744	$30745 = 5 \cdot 11 \cdot 13 \cdot 43$
(223, 2437, 13071, 18166, 20825)	54721	54720	54721
(223, 4211, 9087, 22606, 31915)	68041	68040	68041
(226, 2245, 12123, 16837, 19307)	50737	50400	$12769 = 113^2$
(226, 3143, 6735, 16837, 23797)	50737	50400	$12769 = 113^2$
(227, 721, 856, 2523, 4099)	8425	8424	$8425 = 5^2 \cdot 337$
(227, 901, 1051, 3228, 5179)	10585	10584	$10585 = 5 \cdot 29 \cdot 73$
(227, 1015, 3496, 4737, 9247)	18721	18720	$18721 = 97 \cdot 193$
(227, 1241, 4400, 5867, 11507)	23241	23240	$23241 = 3 \cdot 61 \cdot 127$
(229, 2503, 5461, 13652, 19341)	41185	41184	$41185 = 5 \cdot 8237$
(235, 323, 334, 891, 1459)	3241	3240	$3241 = 7 \cdot 463$
(237, 275, 766, 1021, 2023)	4321	4320	$4321 = 29 \cdot 149$
(238, 1301, 7451, 10289, 11827)	31105	31104	$31105 = 5 \cdot 6221$
(241, 639, 3275, 4792, 5671)	14617	14616	$14617 = 47 \cdot 311$
(242, 385, 409, 1419, 2045)	4499	4896	$167281 = 409^2$
(243, 457, 2339, 3280, 3979)	10297	10296	$10297 = 7 \cdot 1471$
(245, 2434, 5355, 13387, 21175)	42595	46776	$1481089 = 1217^2$
(247, 292, 799, 1583, 2121)	5041	5040	$5041 = 71^2$
(247, 1351, 3439, 8474, 12159)	25669	27000	$1825201 = 1361^2$
(247, 3190, 6871, 17177, 27237)	54721	54720	54721
(250, 275, 393, 917, 1441)	3275	3168	$625 = 5^4$
(250, 1367, 3231, 8077, 11557)	24481	24480	24481
(251, 561, 592, 1963, 3115)	6481	6480	6481
(253, 311, 1096, 1377, 2725)	5761	5760	$5761 = 7 \cdot 823$
(253, 1507, 6530, 9795, 17831)	35915	39168	$10660225 = 5^2 \cdot 653^2$
(255, 844, 1519, 4135, 6497)	13249	13248	13249
(259, 285, 407, 950, 1615)	3515	3456	$1369 = 37^2$
(259, 643, 1993, 3536, 6171)	12601	12600	12601
(261, 287, 410, 957, 1653)	3567	3520	$1681 = 41^2$
(261, 491, 1762, 2773, 4795)	10081	10080	$10081 = 17 \cdot 593$
(262, 291, 331, 883, 1475)	3241	3240	$3241 = 7 \cdot 463$
(262, 443, 469, 1641, 2371)	5185	5184	$5185 = 5 \cdot 17 \cdot 61$

Q-Homology 7-Spheres $M_{\mathbf{w},d}^7$ admitting S-E Structures (cont.)			
$\mathbf{w} = (w_0, w_1, w_2, w_3, w_4)$	$d$	$\mu$	Order of $H_3(M_{\mathbf{w},d}^7, \mathbb{Z})$
(263, 1699, 3921, 9802, 15421)	31105	31104	31105 = 5 · 6221
(268, 511, 911, 2599, 3777)	8065	8064	8065 = 5 · 1613
(271, 377, 673, 1696, 2745)	5761	5760	5761 = 7 · 823
(275, 729, 2734, 4465, 7927)	16129	16128	16129 = 127 <sup>2</sup>
(277, 441, 2591, 3748, 4465)	11521	11520	11521 = 41 · 281
(299, 325, 1869, 2492, 3115)	8099	7488	169 = 13 <sup>2</sup>
(301, 363, 1294, 2257, 3851)	8065	8064	8065 = 5 · 1613
(301, 3289, 17342, 24219, 44849)	89999	90000	90601 = 7 <sup>2</sup> · 43 <sup>2</sup>
(301, 3887, 12259, 28704, 44849)	89999	90000	90601 = 7 <sup>2</sup> · 43 <sup>2</sup>
(307, 2441, 13120, 18307, 33867)	68041	68040	68041
(309, 349, 2279, 3244, 3901)	10081	10080	10081 = 17 · 593
(311, 495, 2104, 3403, 6001)	12313	12312	12313 = 7 · 1759
(311, 2473, 8037, 18856, 29365)	59041	59040	59041 = 17 · 23 · 151
(316, 1727, 9577, 13345, 24648)	49612	49770	49856 = 2 <sup>4</sup> · 79 <sup>2</sup>
(316, 2041, 6751, 15857, 24648)	49612	49770	49856 = 2 <sup>4</sup> · 79 <sup>2</sup>
(328, 347, 449, 1571, 2347)	5041	5040	5041 = 71 <sup>2</sup>
(339, 383, 1780, 2839, 4957)	10297	10296	10297 = 7 · 1471
(341, 407, 2306, 3459, 6171)	12683	13824	1329409 = 1153 <sup>2</sup>
(356, 387, 2225, 2967, 5547)	11481	11440	7921 = 89 <sup>2</sup>
(357, 388, 2231, 2975, 5593)	11543	11520	9409 = 97 <sup>2</sup>

#### 4. Discussion of the Table

In this section we give a discussion about the representatives listed in the Table. It is easy to notice the existence of *twins* in the Table. These are rational homology 7-spheres with the same degree  $d$ , Milnor number  $\mu$  and order of  $H_3$ . Twins often occur as adjacent listings with the same  $w_0$ , but this is not always the case as with twins  $d = |H_3| = 10881, \mu = 10880$  and  $w_0 = 101$  and  $109$ , and the twins  $d = |H_3| = 7777$  with  $w_0 = 141$  and  $w_0 = 167$ . Twins may also be members of a larger set, such as the *septuplets* with  $d = |H_3| = 5761$  and  $\mu = 5760$ . These have  $w_0 = 157, 157, 185, 205, 214, 253, 271$ , respectively. Since twins have the same Milnor number, it is tempting to conjecture that twins correspond to homeomorphic or even diffeomorphic links, but we have no proof as of yet. In fact, except for cases where the order of  $H_3$  contains no primes of order larger than one in its prime decomposition, we don't even know that twins have isomorphic  $H_3$ 's. Notice that the order of  $H_3$  tends to be quite large varying from  $169 = 13^2$  to  $17^{12}$  a number over 500 trillion.

Another interesting fact is that of the 184 rational homology 7-spheres listed in the Table, only 10 have even degree, while the remaining 174 have odd degree, and the degree is even if and only if the order of  $H_3$  is even. But even more intriguing is the fact that for all 174 rational homology 7-spheres with odd degree, the order  $|H_3| \equiv 1 \pmod{8}$ . In [BGN4] we construct positive Sasakian structures on homotopy 9-spheres using the rational homology 7-spheres listed in the Table. There we show that the exotic Kervaire sphere can only occur when the degree of the rational homology sphere is even.