

Classification of Universal Decision Elements Using Computer Algebra Systems

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30th Applications of Computer Algebra - ACA 2025

The concept of Universal Decision Elements (UDEs) emerged in the 1950s [1] and underwent further development in the 1960s-70s [2] and [3]. These early efforts, although insightful, were constrained by the limitations of contemporary hardware and lacked exhaustive formal analysis. Here, we revisit the problem from a modern perspective, leveraging the power of Computer Algebra Systems to perform a complete and verifiable classification of all possible UDEs within a clearly defined logical and functional framework.

We introduce a rigorous formalization of the UDE concept, define the precise criteria for their universality, and use symbolic computation to systematically analyze the entire space of candidate logical functions. The resulting classification reveals both known and previously unrecognized universal elements, providing a comprehensive map of the UDE landscape.

Finally, we propose a series of generalizations of the classical UDE framework, extending it to accommodate reversible computing and quantum information processing. By reframing UDEs in terms compatible with reversibility, we contribute to the rapidly evolving field of reversible and quantum computation, and suggest that long-overlooked and largely forgotten constructs from mid-20th-century logic design may acquire new relevance in the emerging paradigms of reversible logic synthesis and post-classical computation.

Keywords: Universal Decision Element

References

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