

Computing centralizers for linear differential operators

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In this talk we are going to present our recent work [2]. In this work, we are devoted on the computation and the study of the centralizer of a linear ordinary differential operator $Z(L)$, i.e., the set of linear differential operators that commute with the given operator L . When the centralizer is non-trivial, it is a known result that $Z(L)$ is the coordinate ring of a spectral curve.

Based on Goodearl's structural result [1] and the concept of almost commuting operators [3], we provide a new algorithm to compute a filtered basis of the centralizer $Z(L)$ as a $C[L]$ -module for solutions of the stationary Gelfand-Dickey hierarchies. We also provide a family of examples for solutions of these hierarchies for operators of order 3, 4 and 5.

All results are implemented in the computer algebra system SageMath [4], within the package `dalgebra`.

This is a joint work with Sonia L. Rueda.

References

- [1] Goodearl, K. Centralizers in differential, pseudo-differential and fractional differential operator rings. *Rocky Mountain J. Math.* 13, 4 (1983), 573–618.
- [2] Jiménez-Pastor, A., Rueda, S. L. Effective computation of centralizers of ODOs. *Preprint, arXiv:2505.01289* (2025).
- [3] Jiménez-Pastor, A., Rueda, S. L., Zurro, M. A., Heredero, R. H., and Delgado, R. Computing Almost Commuting Bases of ODOs and Gelfand-Dickey Hierarchies. *Math.Comput.Sci.* 19, 1 (2025).
- [4] The Sage Developers. *SageMath, the Sage Mathematics Software System (Version 10.5)*, 2024. <https://www.sagemath.org>.