

# An algorithmic problem for Nijenhuis Lie algebras

Chia Zargeh

Modern College of Business & Science, Oman

30th Applications of Computer Algebra - ACA 2025

In this work, we address an algorithmic problem for Nijenhuis Lie algebras. We introduce the concept of HNN-extension for Nijenhuis Lie algebras and employ the Gröbner-Shirshov basis theory for free Nijenhuis Lie algebras to provide an embedding theorem.

The role of the Nijenhuis operator on a Lie algebra has been used in the study of integrability of nonlinear evolution equations in [1]. In this work, we spread the concept of HNN-extension which is an important construction in combinatorial group theory to free Nijenhuis Lie algebras. HNN-extension has been spread to various algebraic structures such as Lie (super)algebras, Leibniz algebras, semigroups, and rings. The following presentation exists for HNN-extension of Lie algebra  $\mathcal{L}$ :

$$\mathcal{H} = \langle \mathcal{L}, t \mid [t, a] = d(a), \text{ for all } a \in \mathcal{A} \rangle, \quad (1)$$

where  $d$  is a derivation map defined on a subalgebra  $\mathcal{A}$  and  $t$  is a new generating letter. We develop this construction to the case of free Nijenhuis Lie algebras. To this end, we recall the theory of Gröbner-Shirshov basis for Lie  $\Omega$ -algebras introduced in [3] and provide a presentation for HNN-extension of free Nijenhuis Lie algebras. As for an application of HNN-extension, we provide an embedding theorem. It is worth noting that HNN-extension provides alternative proofs for known embedding theorems, and used in undecidability of Markov properties (see [2] and references herein).

## References

- [1] I. Dorfman. Dirac Structures and Integrability of Nonlinear Evolution Equations. (Wiley, Chichester, 1993).
- [2] A. Najafizadeh, C. Zargeh On the undecidability of Markov properties for Lie superalgebras. *J. Algebra Comb. Discrete Appl.* 12(1):43-52. 2025.
- [3] J. Qiu, Y. Chen. Groebner-Shirshov bases for Lie  $\Omega$ -algebras and free Rota-Baxter Lie algebras. *J. Alg. Appl.*, 16(2):1750190, 2017.