Mathematical Experiments for Mathematics Majors

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Abstract

At Simon Fraser University I teach a second year course entitled "Computing with Calculus". The course is a required course for all mathematics majors and applied mathematics majors. The prerequisites are an integral calculus course and a first programming course. The course covers one variable calculus, a little bit of multivariate calculus (partial derivatives) and some modelling with first order systems of differential equations. Students attend one lecture (one hour) and one lab (one hour) per week for 12 weeks.

One goal of the course is to get mathematics majors to use a mathematical software package (I use Maple) to perform a variety of calculations for calculus. Obviously, we want the students to be able to calculate indefinite integrals and definite integrals, solve (systems) of algebraic equations, and solve differential equations. We want them to be able to do these calculations both exactly, and numerically.

A second goal is to teach the students to visualize everything they are doing. Maple and Mathematica have a wide range of graphics capabilities. From a simple plot of $f(x)$ to plotting an implicit surface $f(x, y, z) = 0$ to creating field plots for systems of differential equations.

The third goal is to teach the students how to do a "mathematical experiment". The experiment may be to disprove a conjecture, check a formula, find an optimum solution, or generate an animation of a mathematical object. Doing mathematical experiments usually requires programming, hence the programming prerequisite. Indeed the course provides students a first opportunity to practice their programming skills on mathematical problems instead of more computing problems.

In the talk I will share six mathematical experiments (one per assignment) that I've found to be interesting and instructive for students. The first experiment is the prime number race (See [1]). This can be done with a single for loop that loops through the primes and counts how many primes are congruent to 1 mod 4 and how many are congruent 3 mod 4. The experiment is to determine which count win's the race? The 1's or the 3's?

References

[1] Andrew Granville and Greg Martin. Prime Number Races. *The American Mathematical Monthly* **113**(1):1–33, 2018.