

Collaboration with ChatGPT for research and teaching in algebraic combinatorics

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In memoriam Josef Böhm

As soon as a new technology is launched, the world of education checks its abilities and possible pedagogical usages. Since ChatGPT has been launched in 2023, researchers have analyzed its affordances, and also compared them with other technologies [5].

In a recent paper [3], we analyzed the capabilities of AI tools (in our case ChatGPT) as a teaching assistant in Linear Algebra, at senior High-School and undergraduate levels. We presented our exploration and thoughts at ACA 2023 [2].

Currently, we explore ChatGPT's abilities as a true research assistant, this time in algebraic combinatorics. We report about our experience with using artificial intelligence tools to conduct genuine mathematical research. We were surprised by the results, and the collaboration with our research partners benefited greatly from integrating AI tools into our work.

Unlike a physicist or chemist who needs expensive research labs, it is often said that a mathematician needs nothing more than a sheet of paper and a pencil. Nowadays, to the pencil one adds a powerful computer and mathematical software.

Typically, computers are used to perform calculations that were once done by hand. This involves programming, so either you know how to program, or you have someone around who does, and he/she will program for you. The problem with programming is that, with every small change in the properties you are studying, you have to change the code.

AI can solve that problem for you. You simply tell it what you want it to do — just like you would do to a fellow mathematician, and it computes for you.

Sometimes, AI serves not only as a research assistant but also as an expert. We asked it to “read” a central book in algebraic combinatorics and we asked it to explain to us a theorem presented in the book, and we hardly understood

it. After some fine tuning, the AI excelled in explaining to us the context of the theorem and also created some helpful examples, which were not taken from the text.

can you bring an example?

Applying Theorem 3.13.1:

- $S = \emptyset$ (no descents).
- $\beta_L(\emptyset)$ counts the number of linear extensions with descent set $S = \emptyset$, which is 1.

More Interesting Case:

Now consider a different poset P' on $\{1, 2, 3\}$ where $1 < 3$ and $2 < 3$, but 1 and 2 are incomparable.

The linear extensions are:

- $(1, 2, 3)$ — no descents ($S = \emptyset$).
- $(2, 1, 3)$ — descent at position 1 ($S = \{1\}$).

Now:

- $\beta_L(\emptyset) = 1$
- $\beta_L(\{1\}) = 1$

This matches the bijection described in the proof: each linear extension corresponds to a unique chain in $J(P')$, and the descent set matches the labels in the chain where the sequence decreases.

If you'd like a more complex example or further explanation, let me know!

Figure 1: Examples ChatGPT

When you read a mathematical theorem in a book and try to understand it, you may come across concepts you do not know or do not remember. If you have an expert in the field in your vicinity, you can ask him for help. If you are alone, you will have no choice but to flip through the book and find it. Then return to the place you were reading.

Here, the AI can be helpful. When requested to explain to us a notion that had already appeared in the text, it gave us an excellent answer. Apparently, the bot read the book.

In our talk, we will describe the different sessions with the AI, the examples it provided, and how this improved our AI literacy, making the usage of the new technology an integral part of the newly acquired mathematical knowledge (see [1]).

Our personal instrumental genesis with the bot shows clearly both directions of the process: instrumentation and instrumentalization (see [12, 11]).

Moreover, the OECD defined the 4 C's of 21st Century Education: Communication, Collaboration, Critical Thinking and Creativity (see Figure 2).

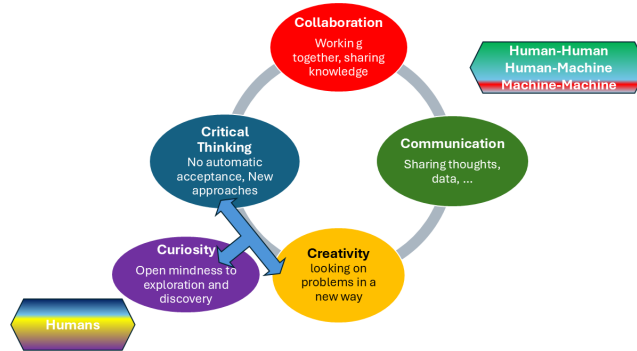


Figure 2: The 4 (or more?) C's of 21st Century Education in a technology rich environment

They meant Communication, Collaboration between humans. In [8, 10], communication is extended to man-machine and machine-machine. Here we experienced new endeavors with man-and-machine. Critical thinking applied for [3] and after, enabled us to develop an efficient dialog with the bot. Communication with the bot is supposed to be in natural language; nevertheless there are some rules to follow in order for it to be efficient and to avoid the strange/false answers we experienced in the past, and still experience in some cases.

We must mention that a 5th C has to be added to the previous list, namely **Curiosity**, without which Creativity may be limited. This issue has been addressed briefly in [9, 7] for plane algebraic geometry and related topics; it is a must for the exploration of the new abilities offered by AI.

In a later step, we will discuss how to apply our explorations and discovery towards a didactic transposition (in the spirit of [6]) and a useful work with students.

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