

GeoGebra Automated Reasoning Tools: a problem from Spanish Civil Service Math Teachers' examination

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GeoGebra is open source software, freely available for non-commercial users. It is dynamic mathematics software that brings together geometry, algebra, spreadsheets, graphing, statistics and calculus in one easy-to-use package, for all levels of education.

In 2013, Bernard Parisse's software Giac* was integrated into GeoGebra's Computer Algebra System view. This allowed to include some automated reasoning tools (ART) in GeoGebra, for mechanically finding relations among geometric elements, for testing the truth or falsity of some statement, for finding additional hypotheses for a given statement to hold, cf. [1], [2]. The algorithms behind these tools are based in computational algebraic geometry, cf. [3].

On the other hand, the Spanish recruitment method to become a civil servant math teacher for the secondary school system requires passing and getting the best grades on a series of exams ("oposiciones"). In one of these recent tests, the candidates were requested to solve an elementary geometry question, asking to conjecture, formulate and, then, to prove, the ratio holding between two particular segments in a given figure (see Fig. 1).

We used GeoGebra ART to accomplish this task, showing, on the one hand, how much it simplifies solving the posed problem; and, on the other, the relevance to adapt and simplify our algorithmic formulation based in elimination ideals [3], to the special zero-dimensional case.

In fact, this example shows that some quite natural, human interpretations of the given situation could lead to a complicated "truth on parts" conclusion (cf. [4]), in which the thesis will simultaneously hold and fail over some irreducible components of the algebraic variety describing the set of instances verifying the hypotheses.

This will imply, in particular, the need to optimize, for the zero dimensional case, the for-

*<https://www-fourier.ujf-grenoble.fr/~parisse/giac.htm>

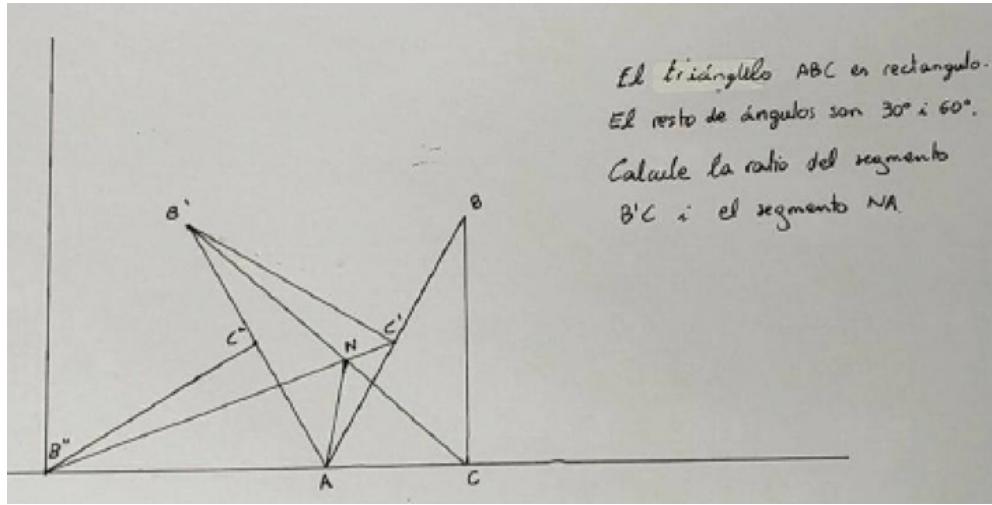


Figure 1: Geometric question from a recent Spanish Math Teachers' recruitment examination: *Triangle ABC is right-angled. The rest of the angles are 30° and 60° . Find the ratio between segment $B'C$ and NA*

mulation of the algorithms for detecting “truth on parts”, thus warning the user about some hidden, unexpected problem that requires further analysis from his/her side.

Our talk will address both the issues related to the mathematical improvements of the automated reasoning algorithms that this example has suggested, as well as the analysis of the desirable interrelation human/machine that could be behind a future scenario towards improving the chances of “passing” the “oposiciones” examination.

Keywords

Dynamic Geometry, Automated Reasoning, GeoGebra

References

- [1] M. A. ABÁNADES, F. BOTANA, Z. KOVÁCS, T. RECIO, C. SÓLYOM-GECSE, Development of automatic reasoning tools in GeoGebra. *ACM Communications in Computer Algebra* **50**(30), 85–88 (2016).
- [2] Z. KOVÁCS, T. RECIO, M. P. VÉLEZ, Using Automated Reasoning Tools in GeoGebra in the Teaching and Learning of Proving in Geometry. *International Journal of Technology in Mathematics Education* **25**(2), 33–50 (2018).
- [3] T. RECIO, M. P. VÉLEZ, Automatic discovery of theorems in elementary geometry. *Revista Matemática Complutense* **23**, 63–82 (1999).
- [4] Z. KOVÁCS, T. RECIO, M. P. VÉLEZ, Detecting truth, just on parts. *Revista Matemática Complutense*, <https://doi.org/10.1007/s13163-018-0286-1> (2018).