Gaussian Elimination with Parameters

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Basic mathematics courses, at all levels, involve _many_ opportunities to include CAS packages. Such systems assist with the preparation of:

- classroom slides/notes,

- individualized homework assignments,

- in-class, randomized quizzes,

- class projects,

- extra-credit, further reading,

- final examinations,

- etc.

In this talk we discuss an aspect which affects all of the areas above, i.e., that of solving Gaussian elimination with parameters, in particular for the teaching of basic, first-semester linear algebra.

Right from the beginning of the semester, students are shown how to perform row reduction. As we know, they need to show that there are either no solutions, one unique solution (and what it is), or an infinite number of solutions (and what they are). Are the standard functions of the available packages prepared to show these three possibilities?

The linear systems are then "complicated" by including input parameters. The students need to continue to solve these systems, and specify, based on the input parameters, which of the three possibilites above pertains. Again, do the standard, available functions supply all of the necessary solutions? As we shall show, not all solutions and special cases are covered.

Three approaches are presented using _Mathematica_ [2], including one which gets back to basic, row reduction. This last one is particularly useful, as it does all of the calculations itself (à la Computer-Based Maths [1]), step-by-step, so the student misses nothing, and is nonetheless not bogged down with a myriad of arithmetic calculations. This leaves more time

for *understanding* the matrix (or individual vectors), as well as applications of solutions of linear systems.

We compare the approaches, demonstrating that some have more satisfying results than others, handling all special cases (and *not* unnecessary ones). We show that one of the approaches delays the need for handling special cases of parameter values along the way, obviating the need for students to recall these special cases until the end.

We end off with applying a final approach to most of the exercises posed in the remainder of the linear algebra course.

References
Computer-Based Maths at `computerbasedmath.org`
Mathematica at `www.wolfram.com/mathematica`