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The importance of being continuously continuous

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We discuss two forms of continuity in the context of integration.

The fundamental theorem of calculus requires that the expression for the integral must be continuous on the interval of interest. Computer Algebra systems, however, do not always co-operate with this requirement. Given an integrand that is continuous on an interval, a computer algebra system may not return an expression that is also continuous on the interval. We show how this can happen, how it can be repaired [1], and speculate on why it has not been.

The other type of continuity refers to continuity with respect to parameters. Consider calculus's most famous integral:

$$\int x^n \, dx = \frac{x^{n+1}}{n+1} \, .$$

When n=-1, this expression breaks down, but a valid integral still exists, namely log(x). This can be regarded as a discontinuity in the parameter n. There are many similar integrals whose standard expressions contain parametric discontinuities. We show how such integrals can be made parametrically continuous [2], and demonstrate a program that does this.

Keywords
Integration, Continuity, Kahanian

References

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