Generalizing the Steiner-Lehmus Theorem using the Gröbner Cover

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Abstract

We present an application of a new method (the Gröbner Cover method, to algorithmically discuss parametric polynomial systems of equations) in the realm of automatic discovery of theorems in elementary geometry. Namely, we describe how the Gröbner Cover is particularly well suited to yield the missing hypothesis for a given geometric statement to hold true. This is achieved by addressing the following problem: find those triangles that have at least two bisectors of equal length. The case of two inner bisectors is the well known, XIXth century old, Steiner-Lehmus theorem, but the general case of inner and outer bisectors has been only recently addressed. We will show how the Gröbner Cover method provides automatically the conditions for a triangle to have two equal bisectors of whatever kind, yielding more insight than through any other automatic method.

Key words: automatic discovering, elementary geometry, comprehensive Gröbner system, Gröbner Cover.
MSC: 13P10, 68T15, 51M04.

*This research was partly supported by the Spanish Ministerio de Ciencia y Tecnología under project MTM2009-07242, by the Generalitat de Catalunya under project 2009SGR1040, and by the ESF EUROCORES programme EuroGIGA - ComPoSe IP04 - MICINN Project EUI-EURC-2011-4306.

†This research was partly supported by the Spanish grant MTM2008-04699-C03-03.