CONSTRUCTING AND ENUMERATING MAGIC CIRCLES AND FRANKLIN MAGIC CIRCLES

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Abstract. There are several variations on magic circles. One famous magic circle was constructed by Benjamin Franklin, circa 1752, which is an arrangement of nonnegative integers in a circular grid consisting of eight concentric annuli and eight *radial* segments. Franklin's circle had many properties, including the standard magic property: that the annular sum and the radial sum equal the same magic number M. Franklin's magic circle is an example of what we call an r-magic 8-circle.

Another similar type of magic circle is what we call a d-magic n-circle, which is an arrangement of nonnegative integers in a circular grid consisting of n concentric annuli and n *diametrical* segments, where the annular sum and the diametrical sum equal the same magic number M.

In this presentation, we discuss some techniques in computational algebraic combinatorics and enumerative geometry to construct and to count these variations on magic circles. We provide a very nice description of their minimal Hilbert basis, which is useful in determining the symmetry operations on magic circles and, consequently, in enumerating *natural* magic circles. Finally, we present the enumerating functions for the Franklin magic 8-circles, the r-magic circles, and the d-magic circles.