SUGAR - A Computer System for SUpersolvable Groups and Algorithmic Representation Theory

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Extended Abstract

Let G be a finite group of order n. By Wedderburn's Theorem, the complex group algebra $\mathbb{C}G$ is isomorphic to an algebra of block diagonal matrices: $\mathbb{C}G \simeq \oplus_{k=1}^{h} \mathbb{C}^{d_k \times d_k}$. Every such isomorphism D, a so-called discrete Fourier transform of $\mathbb{C}G$, consists of a full set of pairwise inequivalent irreducible representations D_k of $\mathbb{C}G$. Concerning these generalized DFTs there are two fundamental computational problems:

(1) How can a DFT of G be *generated efficiently*?
(2) Is there a *suitable* DFT of G which can be *performed efficiently*?

In the case of supersolvable groups given by consistent power-commutator presentations (pc-presentations) Baum and Clausen have presented algorithms for both computational problems which are essentially optimal (see [1],[2],[3],[4]).

This paper presents the computer system SUGAR - **SU**persolvable **G**roups and **A**lgorithmic **R**epresentation theory - which realizes both algorithms, the fast generation of DFTs as well as the fast evaluations of DFTs (FFTs) for the class of finite supersolvable groups. For the first time, this class of generalized fast Fourier transforms is made accessible to applications in fields such as digital signal processing. An overview of the overall system is presented in the Fig. 1. The main features of this system are as follows:

- A library of several thousand consistent pc-presentations of finite supersolvable groups G of different order $N = |G|$ has been constructed. These pc-presentations are the input data for the DFT-generation algorithm. Every pc-presentation needs $O(\log^3 N)$ bytes of storage.
- The DFT-generation algorithm constructs from a given pc-presentation a transversal $\mathrm{Irr}(G, \mathcal{T})$ of $\mathcal{T}$-adapted pairwise inequivalent irreducible representations of G. This can be done in time $O(N \log^2 N)$. The DFT-data (matrices of the representations in $\mathrm{Irr}(G, \mathcal{T})$) are stored in a compressed form (in a data format based on the character graph of G) which only needs $O(N)$ bytes of storage (even though the corresponding DFT-matrix is of size $N \times N$). Furthermore, it turns out that all operations can be done symbolically over the additive group $\mathbb{Z}_e$, where e denotes the exponent of G.
- The evaluation of the DFT (FFT) on a complex signal $f \in \mathbb{C}^N$ as well as the reconstruction of the signal (inverse FFT, IFFT) from its spectral components can be done in $O(N \log N)$. These algorithms are implemented in an iterative way, so that the memory requirements are at most $O(N)$ complex numbers at any time.

All algorithms have been implemented in the programming language C++. Numerous tests have shown that the actual running times reflect very well the theoretical complexity bounds. In addition, based on this computer system, generalized Fourier transforms have been applied to digital signal processing. Using thresholding methods, first experiments of randomized signal compression have been conducted.

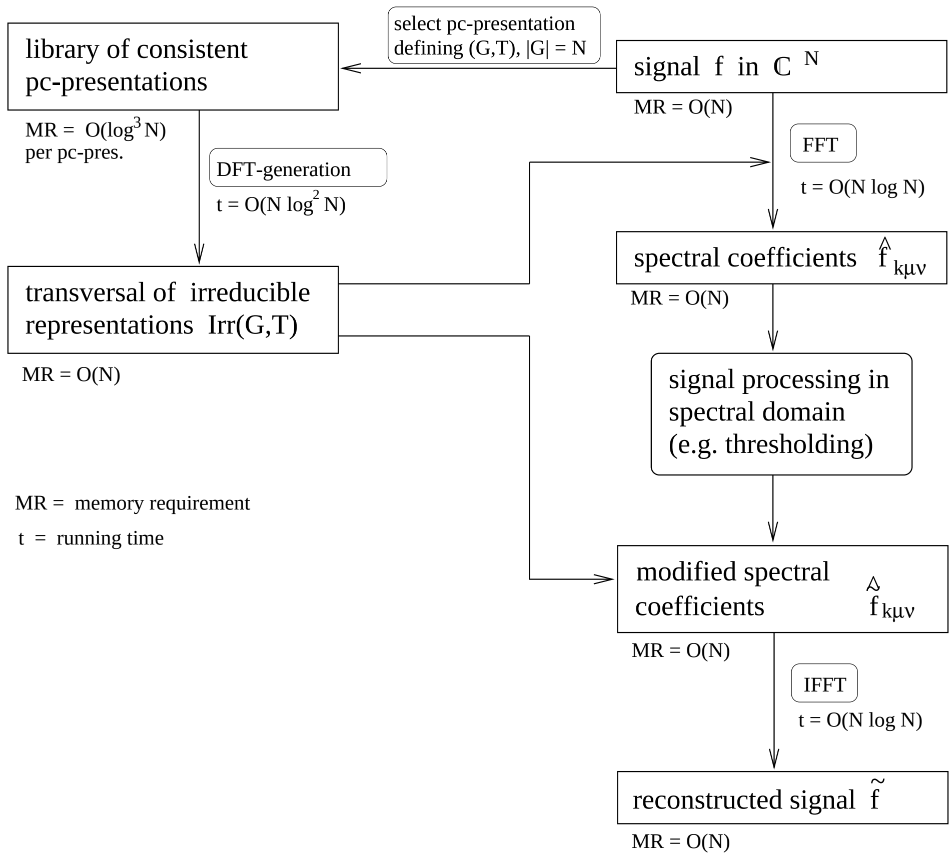

Fig. 1. SUGAR - **SU**persolvable **G**roups and **A**lgorithmic **R**epresentation theory. A computer system for efficient generation and evaluation of generalized discrete Fourier transforms for the class of finite supersolvable groups.

References

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