A Use of Computer Algebra in the Inverse Problem of the Birkhoff-Gustavson Normalization

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abstract

The Birkhoff-Gustavson (BG) normalization has been utilized very effectively in various studies in Hamiltonian systems. For example, for a given Hamiltonian system with a semi-simple stable equilibrium point, the BG normal-form power series of the given Hamiltonian around the equilibrium point provides an 'approximate Hamiltonian system' if truncated up to a finite degree: the approximate system provides a good account of the surfaces of section with sufficiently small energies. The example above implies that finding the class of Hamiltonian systems sharing a certain polynomial in BG normal form amounts to finding a class of Hamiltonian systems exhibiting similar surfaces of sections. *The inverse problem of the BG normalization* has been thereby posed by the speaker [U1,U2]: *What kind of polynomial Hamiltonians can be brought into a given polynomial Hamiltonian in the Birkhoff-Gustavson normal form ?* Since elementary algebraic operations, differentiation, and integration of polynomials have to be repeated many times both in the inverse and the direct problems, computer algebra is worth applying to these problems; the program named 'ANFER' working on REDUCE has been proposed [U1, U2].

The inverse problem of the BG normalization is expected to provide rich subjects related with the integrability of Hamiltonian systems. For example, in solving the inverse problem associated with the two-dimensional perturbed harmonic oscillators with homogeneous cubic-polynomial potentials (PHOCP), the Bertrand-Darboux integrability condition comes out as a necessary and sufficient condition for the PHOCPs to share a BG normal form up to degree-4 with a certain the two-dimensional perturbed harmonic oscillators with homogeneous quartic-polynomial potentials (PHOQP).

The aim of talk is to present

(1) the algorithm of ANFER solving the inverse problem,

(2) how ANFER works effectively in the inverse problem associated with the PHOCPs,

(3) further results in the inverse problem with the PHOCPs obtained from the viewpoint of separability,

not only from the viewpoint of computer algebra but also from that of mathematical analysis: computer algebra and mathematical analysis have been combined very well in the inverse problem, indeed.

References

[U1] Y.Uwano, Journal of Physics **A**, **33**, 6635–53 (2000).

[U2] Y.Uwano *et. al.*, *Computer Algebra in Scientific Computing*, V.Ganzha *et. al.* eds., 441–61 (Springer-Verlag, 1999).