Application of computer algebra systems to construct high-order difference schemes

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In paper presented, in contrast to [1, 2], new approach to application of computer algebra systems (CAS) to constructing high-order difference schemes is stated. Programming language is developed, which is some extension of C++ language and which preserves syntax of MATHEMATICA language. Let us call it C&MATH. Language proposed allows to use functions of MATHEMATICA language (for instance, functions of solving systems of linear algebraic equations, simplification of algebraic expressions, etc.). Computer program-preprocessor is written in CAS MATHEMATICA which converts code written in C&MATH to code in C++ language. Then the code obtained can be compiled as ordinary computer program in C++ language.

Two ways to construct schemes by method of undetermined coefficients are implemented in C&MATH language: as analytical formulae and numerically [3, 4], when for given equation and boundary conditions for each computational node numerical values of difference equations coefficients are found during solving boundary-value problem. In the first way of scheme constructing, program-preprocessor forms system of linear algebraic equations to determine scheme coefficients on given stencils and then solves it. In the second way, only coefficients of the latter system of equations are computed explicitly. Further they are used to find scheme coefficients during solving difference problem. Approach proposed was successfully used to get over huge analytical work concerned with constructing high-order schemes for solving boundary-value problems for elliptic equations in domain with curvilinear bounary. For the first time we succeeded in creation schemes up to 18th order on grids with square and triangle cells inside the domain and with irregular cells near its curvilinear boundary. Application of this approach to Navier — Stokes equations permits to create 4th-order scheme on 5-nodes stencil. Great number of numerical experiments on solving different boundary-value problems by schemes created and comparison of results obtained with tests and with known results of other researches were made. Effectiveness of obtained schemes was shown [3, 4].

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References

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