APPLICATION OF COMPUTER ALGEBRA TO ANALYSIS OF DIFFERENTIAL EQUATIONS SYSTEMS COMPATIBILITY

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One of the fields of computer algebra application is the theory of differential equations system compatibility. Necessity of analysis on compatibility arises, for instance, when performing group analysis of partial differential equations (PDE), or applying the differential constraints method (DCM) [1]. According to Cartan theorem of compatibility theory, after finite number of continuations any system of differential equations can be reduced to involution (compatible system) or to contradiction.

At present, two equivalent compatibility analysis methods applicable to PDE exist: Cartan method and Janet-Spencer-Kuranishi (JSK) method. In applications, analysis on PDE compatibility often leads to huge symbolic computations, to which modern computer algebra systems (CAS) can be applied. ¿From algorithmic point of view methods of analysis on nonlinear PDE systems compatibility are nonlinear, ramified algorithms, which are too complicated for CAS. There are several attempts of theirs realizations. A special case of reduction of one particular system was considered in [2], and a principal ability of using a computer for solving such a problems was shown. We have used different CASs for realization of Cartan and JSK methods. Full charts of algorithms in algebraic terms were written, software inplementation in REFAL and REDUCE systems was made, particular problems of reduction to involution some overdetermined PDE systems, which arise in hydroaerodynamics, were solved [1-7]. This work was partially supported by the Russian Foundation for Basic Research (grants 99-01-00515 and 00-01-00370) and INTAS (proposal 1222).

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