INVOLUTIVE ALGEBRAIC AND DIFFERENTIAL SYSTEMS

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Among the properties of systems of analytical partial differential equations (PDEs) which may be investigated without their explicit integration there are compatibility and formulation of an initial-value problem providing existence and uniqueness of the analytical solution. The classical Cauchy-Kowalevskaya theorem establishes a class of quasilinear PDEs which admit posing such an initial-value problem.

The main obstacle in investigating other classes of PDE systems of some given order q is existence of integrability conditions, that is, such relations for derivatives of order $\leq q$ which are differential but not pure algebraic consequences of equations in the system. An involutive system of PDEs has all the integrability conditions incorporated in it. This means that prolongations of the system do not reveal integrability conditions. Extension of a system by its integrability conditions is called completion. The quasilinear systems of Cauchy-Kowalevskaya type form a particular family of involutive systems.

In this talk we present some recent achievements in development of constructive methods for completion to involution of polynomial and linear differential systems. We argue that polynomial involutive systems give a useful alternative of the reduced Gröbner bases in commutative algebra. The general involutive approach is based on the concept of involutive monomial division which is defined for a monomial set. Every particular division provides for each monomial in the set the self-consistent separation of variables into multiplicative and non-multiplicative. An involutive basis is constructed by combining of non-multiplicative prolongations with multiplicative reductions.

In differential algebra, in addition to their role as canonical bases of differential ideals, linear and quasilinear (orthonomic) involutive systems allow one to pose of an initial value problem providing uniqueness of its solution and to determine the structure of arbitrariness in general analytical solution.

To illustrate this technique by important example we consider the Navier-Stokes equations in R^2 for an incompressible fluid and determine the functional arbitrariness in their general analytic solution.

As another important application of the involutivity we consider the Lie symmetry analysis of nonlinear differential equations and show that by completion to involution of the determining linear PDEs for Lie symmetry generators one can easy find the size of symmetry group without explicit integration of the determining equations.