MORPHISMS AND BINOMIAL IDEALS IN POLYNOMIAL AND DIFFERENTIAL POLYNOMIAL RINGS

Giuseppa Carra' Ferro
Department of Mathematics and Computer Science, University of Catania, Viale Andrea Doria 6 95125 Catania, Italy, e-mail: carra@dmi.unict.it

Binomial ideals in polynomial rings is one of the most studied subjects in the last years, since they are simpler than generic ideals but carry more informations than monomial ideals. Let I be a nonzero ideal in A=$K[X_1,\ldots,X_n]$ and let σ be a term ordering on T_A. Let F=$\{f_1,\ldots,f_r\}$ be the reduced Gröbner basis of I with respect to σ and let $f_i=\sum_{j=1,\ldots,s(i)} c_{ij}t_{ij}+c_{i0}$, $c_{ij}\in K^*$, $c_{i0}\in K$, $t_{ij}\in T_A$ and $t_{ij}\neq 1$ for all $i=1,\ldots,r$ and $j=1,\ldots,r(i)$. Let $m=\sum_{i=1,\ldots,r} s(i)$. Let φ_I be the map from B=$K[Y_{11},\ldots,Y_{1s(1)},\ldots,Y_{r1},\ldots,Y_{rs(r)}]$= $K[Y_1,\ldots,Y_m]$ to A defined by $\varphi(Y_{ij})=t_{ij}$ for all $i=1,\ldots,r$ and $j=1,\ldots,r(i)$. φ_I defines a ring homorphism. Let J_{φ_I}=$(Yij-tij$: $i=1,\ldots,r$ and $j=1,\ldots,r(i))$ in $K[Y_{11},\ldots,Y_{1s(1)},\ldots,Y_{r1},\ldots,Y_{rs(r)},X_1,\ldots,X_n]$. Let τ be an elimination term ordering with $X_h>_\tau Y_{ij}$ for all $h=1,\ldots,n$, $i=1,\ldots,r$ and $j=1,\ldots,r(i))$, such that its restriction to T_A is σ. Let $H(I)=J_{\varphi_I}\cap B$. It is well known that $H(I)$ is a prime binomial ideal and $H(I)=Ker(\varphi_I)$. φ_I defines a polynomial morphism ϕ_I from A^n_K to A^m_K, such that

$$\phi_I(X_1,\ldots,X_n)=(Y_{11},\ldots,Y_{1s(1)},\ldots,Y_{r1},\ldots,Y_{rs(r)}).$$

The closure of $\phi_I(A^n_K)$ in the Zariski topology is a binomial variety, whenever $\phi_I(A^n_K)\neq A^m_K$. The polynomial morphism ϕ_I defines a birational isomorphism between the affine algebraic set $V(I)$ in A^n_K and an affine algebraic set in A^m_K, which is intersection of a binomial affine algebraic set and linear spaces. This result can be extended to the case of differential affine algebraic sets.