Application of Involutive Division Methods to Polynomial and Differential Equations

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Abstract

A new algorithmic technique is presented for constructing of polynomial and differential Gröbner bases of the special form called involutive by analogue with involutive systems of partial differential equations. The corner-stone of the new technique is a concept of involutive monomial division which provides the partition of variables into multiplicative and non-multiplicative ones. This partition leads to a self-consistent procedure for computation of an involutive basis based on non-multiplicative prolongations and multiplicative reductions. Every involutive division generates a particular form of the computational procedure. Along with three involutive divisions used by Thomas, Janet and Pommaret in analysis of partial differential equations we consider some other divisions of practical interest. It is shown that, given an involutive division and an admissible monomial ordering, a minimal involutive basis is uniquely defined for polynomial and linear partial differential equations much as a reduced Gröbner basis.