Quasi-exactly Solvable Spectral Problems for Ordinary Differential Operators

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An ordinary differential operator is said to be quasi-exactly solvable if it is expressible as a polynomial in the generators of a finite-dimensional Lie algebra of first-order differential operators admitting a finite-dimensional module of smooth functions. This Lie algebra can thus be thought of as a hidden symmetry algebra for the given operator. Indeed, if the module is n-dimensional, then one can obtain n eigenvalues of the operator, counting multiplicities, by diagonalizing an $n \times n$ matrix, which is an algebraic problem. Many Schrödinger operators of interest in quantum mechanics are quasi-exactly solvable. Examples include some families of sextic anharmonic oscillators, as well as the Morse, Pöschl-Teller and Mathieu potentials. We will survey the known results on the classification and normal forms for quasi-exactly solvable ordinary differential operators and discuss some open problems. This is joint work with Artemio Gonzalez-Lopez (Universidad Complutense, Madrid) and Peter Olver (University of Minnesota).