Concepts and Algorithms of Computer Analysis for an Approximate Solution of ODE's

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1. Purpose and methodology of Computer Analysis

Whereas in Computer Algebra the dominant object is algorithms based on algebraic concepts and aimed at finding exact solutions, Computer Analysis gives priority to algorithms using CAS for finding controlled analytic approximate solutions. The point is to find efficiently practicable and transparent formula expressions reflecting the qualitative properties of a given problem and securing a desired accuracy. This implies practicable error estimations which must always be performed by the Computer.

2. Algorithms for approximate solutions

In the field of ODEs several algorithms could be developed and tried in the last decade. In accordance with the purpose of Computer Analysis preference is given to mathematical approximation methods allowing an automatic adaptation to a given problem and giving a transparent short formula expression at a reasonable expense. The paper will give a survay of some these algorithms. In recent years good results could be achieved by a two-step concept. In the first step the given problem will be adapted by means of a neighbouring problem which must be suitably chosen and allow to find an exact solution by algorithms of Computer Algebra. Then the fundamental solutions form the basis for an adequate approximating ansatz in the second step. This two-step concept has the advantage of allowing a close problem adaptation. It offers a wide field for the development of several algorithms depending first on the approximation criterions used in both steps and second on the selected family of neighbouring problems. Interpreting the steps as projection methods allows a uniform theoretic foundation.

A modification of this strategy results in an algorithm for the approximate computation of non-exponential solutions for the class of simple differential equations. Based on efficient algorithms for the finding exponential solutions or some subclasses of such solutions a relatively wide class of adapted differential equations has been made available which may serve as a basis for approximations. This class consists of simple differential equations, the relevant differential operators of which are factorizable by differential operators of first order with exponential solutions.

3. Practicable error estimations

For finding error estimations a special procedure has proved its worth. This procedure will allow to completely calculate error boundaries by computer, if the coefficient functions of the differential equations belong to a reasonably restricted function class. It will be necessary to formulate the error function in the form of a fix point equation which will use Green's function of a linear neighbouring differential equation which in its turn can be derived from the given differential operator by means of linearization.

If the non-linear remainder operators satisfy special Lipschitz-relations of second order in surroundings of the approximate solution, fixpoint theorems will give the wanted estimations. In the meantime the principle has been modified and extended. It will be discussed in the paper by taking a non-linear differential equation of n-th order as an example. This procedure shows the advantage of combining the symbolic with the proven numerical computation as a typical attribute of the algorithms of Computer Analysis. The Lipschitz-estimations needed for non-linear differential equations are supplied by a specially developed Lipschitz-calculus.