Point Mapping-Based Analysis of Nonlinear Systems Using Symbolic Algebra

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Symbolic algebra based analysis of dynamical systems has increased significantly in recent years. In this paper, we present an approach for the analysis of stability and bifurcations of periodic solutions of multivariable, multiparameter nonlinear systems described by coupled ordinary differential equations. The formulation is based on the concept of *point mapping* (or period to period mapping). Using computer algebra, symbolic expressions for the point mapping and its dependence on system's parameters are evaluated using a recursive procedure. The symbolic truncation algorithm described here yields coefficients of a multinomial expansion of the point mapping in terms of the state variables and parameters. In this algorithm, the truncation order is arbitrary, and the evaluation of coefficients of the lower order terms is not affected by those of the higher order terms. The symbolic point mapping expression is then used to express the stability and bifurcation conditions as explicit functional relations between system's parameters. The approach is first demonstrated by carrying out a perturbation type analysis for determining harmonic solutions and stability of the Duffing's oscillator. Then the method is used to study the parametric stability and bifurcation of a nonlinear Mathieu's equation. It is shown that, unlike perturbation analysis, the proposed symbolic point mapping method provides a very accurate solution even for *large values* of the parameters. Moreover, unlike existing symbolic computation approaches, the proposed method can be applied to multiple degree-of-freedom systems with many design parameters.