An Algorithm for Constructing Integrating Factors and Corresponding First Integrals for Systems of ODE's

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For first order scalar ODEs, Sophus Lie showed how to construct an integrating factor for each admitted point symmetry; conversely, Lie showed that each integrating factor yields admitted point symmetries.

In general, for systems of ODEs, integrating factors are factors yielding conservation laws. If a system of ODEs is self-adjoint (i.e., its linearized system is self-adjoint), then integrating factors must be symmetries of the system. If a given system of ODEs is not self-adjoint, then integrating factors must be solutions of the adjoint system of its linearized system.

In this talk we present a formulation which yields all integrating factors and corresponding first integrals for any given system of ODEs. The first integrals are obtained algorithmically through an adjoint invariance condition together with a formula which directly yields a conservation law for any solution of the condition. When the given system is self-adjoint, this algorithm is computationally simpler than the classical Noether's Theorem for obtaining conservation laws.

Through an example we will show that our formulation has advantages over Lie's method for reducing the order of ODEs.