Computer Algebra and the Method of Modified Equations

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The method of modified equations is a venerable technique for the analysis and understanding of the performance of numerical methods for the solution of ordinary and partial differential equations. One reason it is not too much used in practice, although it gives useful insight, is the tediousness of the calculations, which have to be done with every differential equation and numerical method combination. We here discuss a simple Maple program which automates the construction of modified equations, given a differential equation and a numerical method we wish to analyze. The basic idea is that we find another, modified, differential equation which is a better model of what the numerical method actually solves.

We give several examples showing that the Maple program can, with little effort, give us good insight into the behaviour of the numerical method. In particular, some of the terms in the modified equation have physical interpretations, which helps the user's intuition.

Our current implementation just uses Taylor series, but work on a B-series version is under way, and we hope by the time of the conference to have some preliminary results with which to compare the two approaches.