Normalizer elements and integrating factors

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Determining an integrating factor for an analytic differential equation $\dot{x} = f(x)$ on an open subset of the plane means finding an orbit–equivalent system $\dot{x} = f^*(x)$ that is Hamiltonian with respect to the standard Poisson bracket. The problem of finding such an integrating factor which is algebraic over the field of rational (resp. local meromorphic) functions comes up in such diverse areas as determining whether a polynomial differential equation has an elementary first integral (Prelle and Singer, 1983), or in connection with normal forms near a stationary point in the analytic case.

There is a correspondence between integrating factors and local Lie groups of orbital symmetries. In fact, it was noted by Lie himself that every nontrivial vector field generating such a group can be used to explicitly construct an integrating factor. It seems to have been little noticed that, conversely, knowledge of an integrating factor gives rise to the construction of a vector field generating a symmetry group.

We will discuss several properties of integrating factors, and also consider the question of characterizing differential equations that admit certain prescribed functions as integrating factors. Results include yet another characterization of local analytic equations that admit a convergent transformation into normal form, and also yield local conditions on the possible existence of