Design of nonseparable bidimensional wavelets and filter banks using Gröbner bases techniques*

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Abstract

Filter banks are important tools of signal and image processing, connected to the mathematical construction of wavelet bases. The issue consists in designing filter banks holding all required properties (orthogonality, symmetries) with maximum flatness. It is known that interesting families of orthogonal filter banks with symmetries are parametrized with polynomial matrices products. The design then consists in calculating these parameters in order to maximize the flatness. We can then turn the problem into looking for *real solutions* of polynomial systems. Direct application of efficient zero dimensional provides some results, but most of the systems have two (resp. four) free parameters. Other Gröbner basis techniques give a change of variables which leads to a much simpler formulation. Computing lexicographic Gröbner basis of the resulting systems (by using an efficient change of ordering algorithm) let us describe explicitely the solutions as parametrized by those parameters. Then numerical methods are used to optimize these free parameters for relevant criteria from signal and image processing points of view. The resulting filter banks and wavelet hold improved characteristics compared to the previous work in this area.

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