Simplification of Truth Invariant CAD's and Solution Formula Construction

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Abstract

Given a prenex formula F of elementary real algebra having k free variables, the CAD method of quantifier elimination produces a truth-invariant CAD, $\mathbf{D}$, for F, that is, a CAD of k-dimensional real space in each cell of which F is truth-invariant, together with an assignment to each cell of its truth value. The method also produces a set $\mathbf{P}$ of irreducible projection polynomials, the real zeros of which form the boundaries of the cells. Typically there exists a coarser truth-invariant CAD, $\mathbf{D}'$, for F and a subset, $\mathbf{P}'$, of $\mathbf{P}$ whose real zeros form the boundaries of $\mathbf{D}'$. We describe an efficient method for deriving such a $\mathbf{D}'$ and $\mathbf{P}'$ from $\mathbf{D}$ and $\mathbf{P}$. The original set $\mathbf{P}$ may not suffice for construction of a solution formula (a quantifier-free formula equivalent to F), but $\mathbf{P}'$ will suffice if $\mathbf{P}$ does. We describe a method for augmenting $\mathbf{P}'$ in this case to a larger set that suffices. In either case the task of solution formula construction is much reduced by the smaller size of $\mathbf{P}'$.