Mathematical Menopause, or, A Young Man’s Game?

My mathematical career was nonstandard. I started graduate school at age 29. From 34 to 50, I produced research, much of it well received. After 50 I could no longer create new mathematics. I popularized and philosophized from then until now, age 72. How

typical or how strange has my story been?

Great authorities warn us, “Mathematics is a young man’s game” (see later, under “Harriety vs. Littlewood”). My starting age for research contradicts the rule, but my concluding age seems to verify it.

Albert Einstein said, “A person who has not read his great contribution to science before the age of thirty will never do so.” [6]

The French-Jewish number theorist André Weil said, “Mathematical talent usually shows itself at an early age. There are examples to show that in mathematics an old person can do useful work, even inspired work; but they are rare and each case fills us with wonder and admiration.”

[13] The Bourbaki collective expulsed members at 50. At his fiftieth birthday party the great German function-theorist Felix Klein whispered to his English student Grace Chisholm Young, said to be his favorite pupil, “Ah, I envy you. You are in the happy age of productivity. When everyone begins to speak well of you, you are on the downward road.” [14]

In a New Yorker article, “Mathematics and Creativity,” Alfred Adler wrote;

… consuming commitment can rarely be continued into middle and old age, and mathematicians, after a time, do minor work. In addition, mathematics is continually generating new concepts which seem profound to the older men and must be painstakingly studied and learned. The young mathematicians absorb these concepts in their university studies and find them simple. What is agonizingly difficult for their teachers appears only natural to them. The students begin where the teachers have stopped, the teachers become scholarly observers.”

[1]
On the other hand, in his biography of the Israeli-American mathematician Abraham Robinson [5], Joseph Dauben wrote,

"[Robinson] was always pleased to dispel the myth that the best mathematicians were under thirty and that a mathematician did her or his best work early, at the very start of one's career. As a striking counterexample, Robinson's best mathematics was only beginning to reap the benefits of his wide experience when, suddenly, at the age of fifty five, he died."

Abraham De Movere (1667–1754) found his presumably most important result when he was 65—by "local central limit theorem". . . De Movere had to stay competitive as a problem-solver in order to attract noble coffee house frequenters as paying clients for instruction. De Movere in old age used to sleep every day a bit longer until the sleeping phase reached 24 hours. (Ivo Schneider, e-mail communication)

Weirsgras was 70 when he discovered polynomial approximation.

The English-Jewish algebraist J. J. Sylvester pointed out that, Lobachevsky, Euler, Laprange, Laplace, Gauss, Plato, Archimedes, and Pythagoras all were productive un-till their seventies or eighties. And of course, we would add Sylvester himself. "The mathematician lives long and lives young," he wrote. "The wings of the soul do not early drop off, nor do its pores become clogged with the earthy particles blown from the dusty highways of vulgar life."

William Glassbrenner Jr.

The Questionnaire

Here are the questions I sent:

1. What opinions or information do you have about aging and mathematicians?
2. How old are you now?
3. How old were you when you started mathematical research?
4. What have been your main fields of mathematical research? How would you compare the value and interest of your research at the beginning of your career and that of your most recent research? How do you think the mathematical community benefits from this?
5. Did you find a certain age that you had lost some zest or drive or facility for mathematical research? At what age? What happened?
6. Did you have such experiences more than once?
7. Do you attribute them to aging or other causes?
8. Did you give up your research work? Did you switch to another field of research? Which other? What was your success? Did you go from pure to applied? Theoretic to numerical? Was the new field close to the old one, or much different?
9. Did you then develop a more intense and committed interest in teaching mathematics? In writing textbooks?
10. Did you collaborate more or less? Did you collaborate with junior, equals, or seniors? (G.C. Rota: "At my age the work of the collaborator is crucial.")
11. Did you develop a more serious commitment to non-mathematical activities? Which ones? Have those activities been able to replace mathematical research for you?
12. In your mature years have you tended to return to the subjects and problems of your youth? (Hilb)
13. Did you feel a strong sense of loss in giving up your earlier research goal?
14. Did those experiences affect your standing in your department? How? By smaller pay raise? By less influence in decision making in your department? In the math profession?
15. Have you suggestions for individuals or institutions to prolong the period of active research?
16. Other questions, comments, suggestions?
One recipient thought my questionnaire was biased toward pessimism—reflecting my own depressed personal- ity. Many respondents praised my project. They called it "refreshing," "provocative," "most worthy." One wrote, "I wish, as probably many people do, that there was something with a little authority written, and I am very pleased that someone of your stature is undertaking it."

Responses trickled in for six months. It took another six months to absorb them and to see a way to present them. There is no claim that my choice of 250 was "typical," let alone "random." And the 66 of 250 who answered are certainly not typical. They are biased toward people who answer questionnaires, who like to hear from an old acquaintance, who are willing to consider some possibly painful issues, and who aren't too unhappy or ashamed of their lot in mathematical life. The people who don't respond to questionnaires are like the dark matter of the cosmos; we know they are out there, but we can only guess what they look like.

Most responses didn't deal directly with Hardy's claim that if you're ever going to do anything important you must do it when you are young. A differential geometer from California pointed out that this isn't the same question as whether you are still active in old age. (Or, as people say nowadays, when you're "older.""

Two of my respondents know of earlier surveys by fa- mous mathematicians. One respondent said George Mackey did a study of 50 leading mathematicians, and concluded that on average their best work was done in their late 30s. Another respondent said Gail Young did a study of people who matured very young in mathematics. He found that they generally burn out early. Young felt there was a fairly constant period during which a person could do very cre- ative work. Some had their period earlier, others had it later. The questionnaire invites recipients to tell as much as they like about their current and past situations. The au-
"As you get older you know too much."

Group 1: No general statement can be made about mathematical aging.

"All generalizations are false, especially this one."

"Better not generalize—we are all different."  
- Probabilitist, British Columbia, age 62

"I met Pólya when he was past 70, and I thought he would go on forever, which have met a number of promising mathematicians who faded before 30."  
- Analyst, Maryland, age 79

"I don't see any general patterns. Some never enjoyed research, some enjoy it but don't want to do a lot of it. On a superficial view these are indistinguishable from people who lose zest at age 35, but the reason is different, when they achieve tenure their research declines, just because the pressure for it is gone."  
- Matrix-theorist, Ontario, age 73

Group 2: Mathematicians are best in youth.

Here we hear of some sad, even tragic experiences.

"One of my old, dear friends suddenly went dry in research at age 40. It was very traumatic for him, and for me to observe."  
- Analyst—applied mathematician, Texas, age 54

"My zest is fine, but capacity much diminished before age 55. Age and alcohol and depression."  
- Analyst, California, age 72

"One does best between age 20 and 50. My most recent research (c. 1906) is not as good as my work in the 1950s."  
- Differential geometer, California, age 67

"At around 55 I have lost whatever originality I once possessed. But not the desire to learn and try."  
- Analyst, Maryland, age 79

"I used to work late at night, but now I'm too exhausted to do more than make calendar entries and clean up my study."  
- Analyst, Louisiana, age almost 68

"Clearly at my age I can't keep up with the best young people. Some old timers have looked foolish in their later research efforts. My hope is at least to avoid that."  
- Applied mathematician, Rhode Island, age 71

"The vast majority of mathematicians do best before 40, and often as not, before 30. But that would be hard to substantiate; one would need to know the life work of a mathematician, and make reliable judgments about it... The best counterexample I know is Legendre, who proved the case n = 5 of Fermat's last theorem when he was in his seventies."  
- Differential geometer, California, age 72

"Men age faster than we girls. It makes up for them being bullies earlier. How to pep them up? I try... People whose lives are fairly stable and satisfactory keep going a lot better. One of my colleagues gave up research at 42 when his marriage broke up. Another similar at 48."  
- Probabilitist, British Columbia, age 68

"As you get older you know too much. You have all these methods and you try all the combinations and variations you can think of. You're running down the old tracks and nothing works." (Vlad Niren, [2])

"Mathematical tools tend to be introspective, with unbalanced expenditure of mental energy. As one grows older there is desire for other forms of expression, which dilutes the intensity to solve problems. "What does it all mean?" is asked more often, which also can slow down progress."

"Aging has two sides—you own age, and the age and aging of your subject and your contemporaries. This aging is brought about by the work of younger competitors."  
- Analyst, Sweden, age 89
"The field of mathematics moves very fast. The pace has been quite extraordinary in the past 50 years. Just trying to keep up in one's specialty requires many hours of effort. One doesn't feel comfortable doing the same old thing. Some great mathematicians have been unable to handle this. When a decent problem comes along which seems accessible, I'm eager to jump in. The trouble is that the frontier is moving so fast. It's not that we give up research mathematics, research mathematics gets away from us." Geometer, California

Group 3: Mathematicians may be as good or better in their later years.

"A Young Woman's Game?", below, provides impressive testimony that women mathematicians are often at their best in their 30s, 40s, even 50s.

"Mordell is reputed to have said modestly, 'I did work in my 70s many a younger man would have been proud to have done.' Among my teachers, I know that Buring, Abihara, Zariski, Mackey worked intensively on research when they were quite old." Analyst, Rhode Island, age 72

"Since I became emeritus in July 1995 my research has increased. Most of it is joint with former students and postdocs. The mathematical tools are ones I've used before—this is probably typical. It is a great relief to shed 9 years of department chairmanship, too many committees, and obligations to seek external grants. I no longer attend department meetings." Analyst-applied mathematician, Rhode Island, age 71

"Some of my best work was done after age 47. Possible motivations were a bad spell of drinking and divorce from 1974 to 1977, and prostate cancer treated successfully by radiation and implants. After such trauma I tend to over-achieve." Analyst, Illinois, age 69

"Knowledge and experience count for a lot more than CPU speed. At the minimum it improves your mathematical taste. My recent papers are a lot stronger than those just after the Ph.D." Analyst, Alabama, age 61

"Young guys may luck out but often only when some one older points the way." Applied mathematician, Colorado, age 64

"The young may find gold but cannot read the land; the older have familiarity with the landscape, which guides them to where to dig." Recently a friend compared me with Brahms, who turned out great works throughout his life! I hope to live up to the praise." Numerical analyst, Ohio, age 70

Group 4: Symptoms and strategies.

By this heading I mean symptoms of aging, and strategies to cope with it.

"My wife and I have been happily married for forty-four years; that's extremely important. Our garden takes a major part of our time in the growing season." Applied mathematician, Rhode Island, age 71

"My memory is not what it used to be. My work takes much longer and the need for careful notes is greater. My best work was around age 40." Analyst, Sweden, age 68

"My drive for research and my direction haven't changed dramatically. I just don't think as clearly and quickly now. But I've grown efficient in other respects, and as a result my best work has been in the last 15 years. I collaborate more, but still do a lot by myself." Applied mathematician, Utah

"As I age my memory declines, making it more difficult to keep in mind all the threads of a complicated situation. Also my computational abilities decline—I take longer to get through a routine calculation, and make more mistakes.

A Young Woman's Game?

Does the slogan "mathematics is a young man's game" exclude women? Or should it be "a young person's game"? Claudia Henzen [8] says no. "...there is a deeply engrained belief that mathematics is a 'young man's game,' despite the fact that there is no compelling evidence to support this hypothesis; instead, the studies that have been done suggest the contrary. But when the image and reality differ, it is often the image that can have a more powerful influence on attitudes, practices, and policies. If the focus were not so much on the young, white mathematicians, it would be easier to design programs with women in mind. For example, [recognizing] the fact that women are likely to have children in what is traditionally considered their prime mathematical years...looking at their productivity over a longer time span...recognizing that women may need to enter the mathematical research pipeline later in life, so Joan Birman did, or they may need to work part time for a period to balance having children with mathematics research, as Mary Ellen Rudin did."

The president of the Mathematics Association for Women, Marjorie Frei, told Henzen, "I've never felt that you're over the hill if you're in your late thirties. I think I've done my best work later on, by a long shot."

The leading trend-theorist Joan Birman focused better on work after the birth of her children were settled, her children were older, etc. "I think doing mathematics when you're enthusiastic is important—not your age."

Richard, a famous topologist, said, "I don't think most people's best work will be done by the time they're thirty, and certainly my best work wasn't done until I was fifty-five years old."

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I catch mistakes by my sense of what seems right, rather than by repeating computations. On the other hand, I'm more cannny in developing effective research strategy, and more daring in carrying it out. . . I have an intellectual home with a small but active worldwide community of scholars with similar interests." Applied mathematician, Age 70.

"I try with retiring at the end of this year. I am nervous about it, but clearly recognize the diminishment of my ability to do first-rate research. The main cause is inability to stick with messy detailed manipulations. In the past I could calculate for hours, but now I shy away from such grunge. I still have plenty of things to work on, but I pick them more carefully." Numerical analyst, California, Age 79.

"Getting old is a pain. I still do decent mathematics. However, what I do is very much related to my previous work. I do not jump into a new field, because I have not the same intution as earlier to 'know' it will lead to something. Everything takes much longer to complete and I make more mistakes, or better, I do not know immediately when the result is wrong. So I have to check much more carefully. I have been a good thesis advisor, which I enjoyed very much. Former students still speak to me, and I still work with them. But I have no students any more, because I cannot be sure I will be around in 4 years. Also, young people should work with young people on 'modern' problems. There can be one advantage with old age. If one is lucky and in balance with oneself, one can look at the world as an independent observer." Numerical analyst, Sweden, California.

"The principal obstacles to continuing research are: (a) Research requires energy, and this is in increasingly short supply. (b) Research requires keeping up with the literature, and this becomes difficult as one's mental and physical energy declines. (c) Good research requires breadth and flexibility, but the tendency as one ages is to concentrate on a narrow path, dominated by what one has always done, and knows well.

"Collaboration is essential in maintaining research activity. I have tended to collaborate with juniors, since very many of my collaborators have been my students. The younger partner provides energy and awareness of what is currently a 'hot topic'; the older provides perhaps greater familiarity with the history of the topic and a larger history of available methods." Topologist, New York, Age 76.

"There are many useful things someone with mathematical ability can do. But education and research rewards do not encourage people to branch out and explore. They get stuck in the frontiers of their narrow specialty. The going gets rough when they no longer have the ability or willingness for the concentrated effort to do really complicated technical work. I am still able to do this if I get away for a couple of weeks, but at home commitments to family and work preclude that concentration. It does get harder as you get older, from aging but also from accumulation of other responsibilities and interests." Logician, Indiana, Age 57.

"My enthusiasm for research increased rapidly from 25 to 35 and stayed high for 10 years. During that time I could not imagine doing research. Then I became interested in research in undergraduate mathematics education (RUME). I tried to work in both fields, but my interest and ideas for work in functional analysis disappeared. There are two possible factors. Interest in RUME may have driven out interest in functional analysis. The other was a feeling that I might do other work as good as my best, but I could not do much better." Math education researcher, South Carolina, Age 64.

"My success and pleasure at research is tied to my ability to travel worldwide and make connections with people from diverse cultures. Politics has been extremely important for me to keep my balance, and this is much easier in Canada than the U.S. I have strong human rights interests related to indigenous peoples in North America and the

Some Numbers

Claudia Henrich ("A Young Woman's Game?"") reports an article on young mathematicians in the research literature (as opposed to the anecdotal literature). Thanks to Judith Grabiner for this reference.

Stene's mentor Stephen Cole [4] studied chemists, geologists, physicists, psychologists, and sociologists, and found, "There are basically no differences in the quality of work published by scientists between the ages of 30 and 50. Scientists over the age of 50 are slightly less likely to publish high quality research."

Stene extended Cole's work to mathematicians. She counted publications of 403 "randomly chosen" mathematicians at Ph.D. granting institutions, and sorted them by age. Since attendance of a paper should roughly measure mathematicians' interest in it, she also counted their citations. The numbers in parenthesis in the table that follows are the numbers of sample mathematicians in each age group.

<table>
<thead>
<tr>
<th>Age</th>
<th>Mean number of publications</th>
<th>Mean number of citations</th>
</tr>
</thead>
<tbody>
<tr>
<td>30-35</td>
<td>6.12</td>
<td>2.73 (105)</td>
</tr>
<tr>
<td>36-40</td>
<td>7.50</td>
<td>3.80 (86)</td>
</tr>
<tr>
<td>41-44</td>
<td>8.24</td>
<td>5.14 (93)</td>
</tr>
<tr>
<td>45-49</td>
<td>9.49</td>
<td>5.44 (88)</td>
</tr>
<tr>
<td>50-54</td>
<td>6.32</td>
<td>5.85 (75)</td>
</tr>
<tr>
<td>55+</td>
<td>6.11</td>
<td>5.09 (58)</td>
</tr>
</tbody>
</table>

Stene concludes: "The claim that younger mathematicians (whether for psychological or sociological reasons) are more creative and do more important work is unsubstantiated. I have found no clear relationship between age and achievement in mathematics."
Group 5: Penalties for aging or for following one's path

There are really two topics here. Penalties of either type present an opportunity for some soul searching by mathematicians and organizations.

"As I get closer to retirement I am consulted less about departmental matters." Numerical analyst, California, age 74

"For sure I have lost influence in the department and got (not deserve) tiny raises." Analyst, Louisiana, age almost 62

"Smaller pay raises. Loss influence. No NSF grants. The majors protect themselves. I'm treated like a half breed in the West." Analyst-applied mathematician, Colorado, age 64

"My recent work is more interesting and valuable. Math community isn't interested. Ecology community is." Applied mathematician, British Columbia, age 65

"I attached equal value and interest to all my research. The mathematical community attached little of either to any of it." Analyst-historian, Washington State, age 73

"I have been treated well. I still have my office, 10 years after retirement." Measure theorist, California, age 80

"My department has treated me well. I still have an office and they pay me a small amount for looking after some graduate students. My research is worth more to me than to the department, so there is no strong reason why they should actively support it." Numerical analyst, Sweden, age 66

"By following my bliss, I gave up my opportunity to get to full professor. My most valuable professional achievements are not appreciated by the leadership in my department." Age 62

"It was ignominious that the Fields committee turned down Wales's achievement on account of his age. There is no formal constraint on age for the medal."

"I use knowledge and experience from early areas in learning new things. Shifting fields this way causes one to lose NSF support. This can be replaced after a few years, but I get sick of being funded in new 'lives' and stopped applying. The judgment of proposals is silly: many people simply propose what they have already done and try to guess what directions are politically correct. I found many colleagues singularly narrow in focus and rigid in their approach to scientific discovery. It was often difficult to get anyone to look at my papers." Analyst, Illinois, age 69

"At 28 I had built enough research reputation to do some writing without serious loss of status in the research community. At times I believe the only criterion department chairmen go by is bringing in contracts and grants. At times I think my colleagues regard me as an eccentric anomaly pursuing non-standard paths." Analyst-author, Rhode Island, age 77

"The mathematical community lost interest in my work when fashions changed and I didn't. After a period as chairman, when I was 40, I lost influence in the department." Analyst, California, age 72

"My best paper was never referred to in the later literature. I tell myself this shows it said the last word on its subject." Numer-theorist, Minnesota, age 62

"My chief gain has been realizing that my creativity was very limited—I could not conceivably become a significant mathematician. My seniors had higher expectations for me than were finally justified. I have always had an intense passion for music, and this comforted me." Analyst, Maryland, age 79

"I did feel a loss when what I was doing was not valued by the mathematical community. It took a while for me to value it for its own sake." Liptician, Indiana, age 57

"The value of my research is quite high, the interest by others is quite low. The math community doesn't pay attention to most mathematician's work. I am called on to do diplomatic or administrative jobs. I am not a very able administrator, but compared to the great majority of mathematicians, I am an administrative genius." Applied mathematician, Alberta, age 60

"Some of my best research has been in recent years, yet I have been getting smaller pay raises and have less influence in my department. The situation of some of my contemporaries is even more egregious. Mathematicians departments and organizations don't pay attention to the older members of the profession. My department treats our retirees shabbily: we give them a 'gold watch' when they retire, then forget about them."
"... a big issue is why retirement is synonymous with severance from all academic activities."
**My own comments:**

After retirement I asked for a floppy disk, and the person who was then my department administrator told me (even though I was still a part-time instructor), "You're putting me in an embarrassing position. You're not in the budget. You're-emeritus now. Why don't you just run over to the book store?"

Shabby treatment of aging professors is not special to mathematics. After retiring from the Columbia economics department William Vickrey got a Nobel Prize for work on transportation economics. A New York Times reporter found him in a tiny office far from his department. Vickrey was grateful that after retirement Columbia allowed him any office at all. Perhaps after being written up in the Times he would have been granted a better office, but, sadly and unexpectedly, he died a few days later. (See also Littlewood, in Box 1.)

Sometimes emeriti are even dropped from the department e-mail announcing about seminars, hiring, promotions, retirements, and anything else interesting that is going on.

Yet departments can always use extra hands. Undergraduate advising is often understaffed. Has anybody asked, "Are there emeriti who enjoy advising?"

If there is no librarian on duty in the math library, is there an emeritus who would serve?

There's always too much committee work. Is there an emeritus with years of service on the undergraduate committee or the master's exam committee? Might he/she have something to contribute there?

**Summary**

The responses were very varied. But these five statements would be generally accepted:

1. There's tremendous variation in how mathematicians age. No one pattern describes everybody.
2. Many mathematicians have been productive in advanced age.
3. To most (not all!) mathematicians, aging brings losses in memory and computing ability. These may be compensated by broader perspective and mature judgment. Potentially more serious is slowness or difficulty in learning new material. Some responses were more specific.
4. Live healthy and follow your own bent, not the pressures of others.
5. Older and retired mathematicians are an under-utilized resource for the mathematics community.

Until we find a consensus about which advances are "major," we can't refuse Hardy's claim that no major advance has been made by a mathematician over 50. But his slogan, "Mathematics is a young man's game," is misleading, even harmful. So far as it may discourage people from mathematics when they're not longer young, it's an unplanned and destructive.

**REFERENCES**