1. **Behaviour of Fourier coefficients and trigonometric interpolants.** For the 5 functions $f_1(x) = \sin x$, $f_2(x) = 1/(2 + \sin x)$, $f_3(x) = x$, $f_4(x) = |x - \pi/2|$, $f_5(x) = \sqrt{x}$ defined on $[0, 2\pi]$

(a) Sample the function at $x_j = 2\pi j/N$, $j = 0, \ldots, N$ (choose $N$). Plot the data $(x_j, f(x_j))$

(b) Find the Fourier coefficients $\hat{f}_k$ and plot their absolute value on a log log scale. Discuss the different behaviour of these coefficients.

(c) Plot the trigonometric interpolant $f(x) = \sum \hat{f}_k e^{ikx}$ by sampling it at $M$ equally spaced points, $M > N$. Use the fft and ifft for this, as discussed in class. Discuss the results. (For example, how does the error between the interpolant and the actual function behave? Why?)

2. **Music compression.** A 5 minute song on a CD (sampled at 44,100 samples per second) consists of approximately 50 MB of data. An Ipod with 256 MB of memory can store 5-20 songs of that size. Clearly it is advantageous to truncate the data in some form. Here you should discuss 3 different possible approaches and explain what the problems are with them and why they would not work. The goal is to understand the difficulty of this problem.

(a) Take the Fourier transform of the whole set of the data and apply a bandpass filter in which you set small coefficients (in a certain band) to zero.

(b) Take a block of the data, set all other values to zero, take the Fourier coefficients of the result, and apply a bandpass filter to it.

(c) Take a block of data and multiply by a smooth function that is equal to 1 within the block and 0 outside the block, and smooth in some transition region. Take the Fourier coefficient of the result, and apply a bandpass filter to it.