1. In class you wrote a parallel code that computes \( \int_0^1 \frac{1}{1+x^2} \, dx \) using the trapezoid rule using \( n \) intervals. Plot the speedup and the efficiency as a function of the number \( n_p \) of processors used, for \( n_p = 1, \ldots, 30 \), using \( n \approx 10^4, 10^6, 10^8 \) points. Interpret your results.

2. Compute latency and bandwidth for Hammer, using the timings to send a vector from one process to another using the ring method with blocking sends/receives.

   (a) Use 2 processors (on distinct nodes, using ppn=1 option). To check which nodes you are using, enter a line "cat $PBS_NODEFILE" in your batch.job file. The nodes will be listed in your .o file. If no node is repeated you can be sure you will run job on distinct nodes.

   You need to
   - find the time it takes per message of size \( s \) sent or received. You need this time for various messages sizes \( s \) ranging from 1 byte to 10\(^7\) bytes. Choose values of \( s \) equally spaced on a logarithmic scale (eg \( s = 2^j \times 8 \) bytes, \( j = 0, \ldots, 20 \)). Send and receive a message of this size using the ring method many times and compute the average time per message. The number of times should be big enough (100? 1000?) that your average value remains unchanged if the number of times is increased.
   - plot the time vs \( s \) on a log log scale.
   - find the best least squares fit to the data by a curve of the form \( t = l + s/b \). Plot the least squares fit on a log log scale in the same figure as the data. \( l \) is the latency, \( b \) is the bandwidth. (Alternatively, find the best "eyeball fit".) Compare your values with the values for Infiniband of latency =1.3 microseconds, bandwidth 950 MB/sec.

   You need to submit your code used to obtain the timings in your report.

   (b) Using 24 processors. Obtain the timings, plot \( t \) vs \( s \) and compare with the least squares fit obtained from (a).

   (c) Discuss your results. How many flops can be computed on Hammer (2 GHz processors) in the same time as it takes to send a zero byte message?

3. Compute the evolution of the periodic vortex sheet with initial conditions

   \[
   x(\Gamma, 0) = \Gamma + 0.01 \sin 2\pi \Gamma, \quad y(\Gamma, 0) = -0.01 \sin 2\pi \Gamma
   \]

   up to time \( t = 2 \) using the parameters given below in (a,b). (These parameters are chosen so that the main features of the solution have converged in \( N \) and \( \Delta t \).) For part (a) a serial code is sufficient. For part (b) you need to write the code using both blocking and nonblocking sends and receives.

   (a) \( \delta = .2, \quad N = 500, \quad np = 1, \quad \Delta t = 0.05 \) Plot the solution at the final time.

   (b) \( \delta = 0.1, \quad N \approx 6400, \quad \Delta t = 0.01 \)

   - Plot the solution at the final time.
   - Compute and plot scalability and efficiency of code using 1 \( \leq np \leq 30 \). (Use \( N \) close to 6400 so that it is divisible by np).
   - Compare the runtimes using blocking and nonblocking sends and receives.

   You need to submit the blocking and nonblocking codes in your report.