This pdf file lists daily homework problems, consisting of

*Level 1 problems:* strengthen the basics, often short answer (do these first)
*Level 2 problems:* apply the basics and build understanding (collected and graded)
*Mixed Review:* mixed review of earlier material, often at a slightly more challenging level since you are assumed to already have mastered the material. These are good exam problems.

Quizzes will consist of problems similar to the previous week’s homework. Your instructor will try to put a problem of each type (level 1, level 2, mixed review) on each quiz, *as well as problems that are worked out in the corresponding sections in the textbook.* Exams will consist of problems very similar to those in the homework.

You must work on all problems on a daily basis for a chance to succeed in this class. Additional practice problems with answers are given by the odd problems in the book. Solve all problems without using a calculator, unless specified, as you will not be able to use a calculator on quizzes and exams. You can get any help you are comfortable with, but ultimately, you need to write out the complete solutions on your own, without referring to notes/book/web/tutor.

**HOMEWORK DAY 1  —  Inverse functions**

**Reading:** §6.1.

**Level 1:** §6.1: 3,15,21,24

**Level 2:** due Friday Aug 25

1. Use the Inverse Function Property (Equations 4, p 386) to check whether \( f \) and \( g \) are inverses of each other.

\[
f(x) = \frac{6-x}{7}, \quad g(x) = -7x + 6
\]

2. (NEW) Show that the points \((a, b)\) and \((b, a)\) are symmetric about the line \(y = x\).

2. Use the given graph of \( f \) to answer the following.
   
   (a) Why is \( f \) one-to-one?
   
   (b) What are the domain and range of \( f^{-1} \)?
   
   (c) Find \( f^{-1}(1) \) and \( f^{-1}(0) \).
   
   (d) Copy the graph of \( f \) on your paper and in the same plot, sketch the graph of \( f^{-1} \).

3. For each of the following functions:
   
   (i) Sketch a graph of \( f \) and determine if it is invertible.
   
   (ii) If invertible, find a formula for \( f^{-1} \).
   
   (iii) If invertible, add the graph of \( f^{-1} \) in the same plot in (a) showing the graph of \( f \).

   The graph should clearly show all ranges and domains.

   (a) \( f(x) = x^2 - 2x \)  
   
   (b) \( f(x) = x^2 - 2x, \ x \geq 1 \)  
   
   (c) \( f(x) = 1/x \)  
   
   (d) \( f(x) = x^{1/3} \)

4. (a) Suppose \( f \) is differentiable and invertible for all \( x \in D \), with \( f'(x) \neq 0 \). Derive the formula in Theorem 7, page 388, as follows. Start with the relation

\[
f(f^{-1}(x)) = x
\]

and differentiate both sides with respect to \( x \). Solve for \((f^{-1})'(x)\). What is the geometric meaning of this formula?

(b) Suppose the function \( f \) is invertible, with point \( P(2, 3) \) on its graph, and slope 1/7 at \( P \). Find an equation for the line tangent to \( f^{-1} \) at \( Q(3, 2) \).
5. Answer the following by inspection. That is, without finding an explicit formula for $f^{-1}$ first. (These problems often show up as “quick answer” problems on quizzes and exams.)

   (a) If $f(x) = x^5 + x^3 + x$, find $f^{-1}(3)$, $f(f^{-1}(2))$, $(f^{-1})'(3)$.
   (b) If $f(x) = x^3 + 3\sin(x) + 2\cos(x)$, find $(f^{-1})'(2)$.
   (c) If $f(x) = \frac{x^2}{\sqrt{1+t^3}}\,dt$, find $f^{-1}(0)$, $(f^{-1})'(0)$, equation for tangent line to $f^{-1}$ at $x = 0$.

Mixed Review:

R1.1. Diagnostic Test A (Stewart, p xxiv): 1,2,4,8(e,g),10

R1.2. Sketch a graph of the following polynomials, solely using roots, symmetry, degree and sign of the leading coefficient.

   (a) $f(x) = (x^2 - 1)^3$
   (b) $f(x) = x^3 - cx$, $c > 0$
   (c) $f(t) = 10t - 1.86t^2$
   (d) $v(r) = r_0r^2 - r^3$, $r_0 > 0$
   (e) $F(r) = GMr/R^3$, $G, M, R > 0$
   (f) $F(R) = GMr/R$, $G, M, r > 0$

R1.3. Evaluate

   (a) $\int_{1}^{9} \frac{\sqrt{u} - 2u^2}{u} \, du$
   (b) $\int x(2x + 5)^8 \, dx$
   (c) $\int_{0}^{\pi/8} \sec(2\theta) \tan(2\theta) \, d\theta$
   (d) $\int_{0}^{3} \sqrt{x^2 - 4} \, dx$
   (e) $\int_{-1}^{1} \frac{\sin x}{1 + x^2} \, dx$

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**HOMEWORK DAY 2 — The exponential function $a^x$**

**Reading:** §6.2.

**Level 1:** §6.2: 2,17,31,35,79,81.

**Level 2:** due Friday Aug 25

1. Find the domain and intercepts of the following functions: (a) $f(x) = \frac{1 - e^{1+x^2}}{1 - e^{1-x^2}}$
   (b) $f(x) = \frac{1}{1 + e^x}$
   (c) $f(t) = \frac{e^t}{1 + e^t}$
   (d) $f(t) = \sqrt{1 - \frac{2^t}{1+e^{2t}}}$
   (e) $f(t) = \sin(e^{-t})$

2. In one plot, sketch the graphs of the following functions:
   $f(x) = e^x$, $f(x) = e^{-x}$, $f(x) = 8^x$, $f(x) = 1/8^x$.

3. In separate plots, sketch the graphs of $y = 10^{x+2}$, $y = 1 - e^{-x}/2$, $y = e^{|x|}$.

4. Find the following limits.

   (a) $\lim_{x \to \infty} (1.001)^x$
   (b) $\lim_{x \to -\infty} (1.001)^x$
   (c) $\lim_{x \to \infty} \frac{e^{2x} - e^{-2x}}{e^{2x} + e^{-2x}}$
   (d) $\lim_{x \to -\infty} \frac{e^{2x} - e^{-2x}}{e^{2x} + e^{-2x}}$
   (e) $\lim_{x \to \infty} \frac{2^x - 2^{-x}}{2^x + 2^{-x}}$
   (f) $\lim_{x \to -\infty} \frac{(1/2)^x - (1/2)^{-x}}{(1/2)^x + (1/2)^{-x}}$

5. Differentiate the given functions. §6.2: 32,36,42

6. Integrate the given functions. §6.2: 84,86

7. A drug response curve describes the level of medication in the bloodstream after a drug is administered. A surge function

   $$S(t) = Ate^{-kt}, \quad t \geq 0$$

is often used to model the response curve, reflecting an initial surge in the drug level and then a more gradual decline. The parameters $A, k$ are positive and depend on the particular drug.
(a) Find the intervals where $S$ is increasing, and where it is decreasing.
(b) Use your result in (a) to specify the time and the value of maximal response.

Mixed Review:
R2.1. Find the derivatives of the following functions

(a) $y = x^2 \sin(\pi x)$  
(b) $F(R) = GMr/R$  
(c) $y = \int_2^x \frac{t}{1 + t^3} \, dt$  
(d) $y = \int_1^x \cos(t^2) \, dt$

R2.2. Evaluate

(a) $\int_2^1 \left( 1 + x^2 \right) \, dx$  
(b) $\frac{d}{dx} \left[ \int_1^2 \frac{1}{1 + x^2} \, dx \right]$  
(c) $\frac{d}{dx} \left[ \int_1^2 \frac{1}{1 + t^2} \, dt \right]$.

HOMEWORK DAY 3 — Logarithmic functions

Level 1: §6.3: 3,4,7,8,16,17,18,27,29,45,47.

Level 2: due Friday Sep 1
1. Sketch the graphs of the following functions.
   (a) $f(x) = \ln(1/x)$  
   (b) $f(x) = \ln(x - 1)$  
   (c) $f(x) = \ln(x + c)$, $c > 0$

2. Solve the following equalities and inequalities for $x$
   (a) $\ln x < 0$  
   (b) $\ln x > 1$  
   (c) $\frac{x}{1-x} = 0$  
   (d) $\ln(e^x - 3) = 2$

3. §6.3: 46 (inverse of $Q(t)$)
4. §6.3: 49,50,52 (evaluate limits)
5. §6.3: 65 (find where $f$ is increasing)

Mixed Review:
R3.1. For what values of $r$ does the function $y(t) = e^{rt}$ satisfy the equation $y'' + 6y' + 8y = 0$?
R3.2. Let $f(t) = e^t/(1 + e^t)$.  (a) Find the domain of $f$.  (b) Find where $f > 0$.  (c) Find where $f$ is increasing.  (d) Find $\lim_{t \to \pm \infty} f(t)$.  (e) Use your results to sketch a graph of $f$.

HOMEWORK DAY 4 — Derivatives of the logarithm

Level 1: §6.4: 2,3,6,7,11,21,71,73,77.

Level 2: due Friday Sep 1
1. §6.4: 17,18,20,74,76,78,80 (derivatives and integrals)
2. §6.4: 46, 50 (logarithmic differentiation)
3. §6.4: 61 (find intervals of concavity and inflection points)

Mixed Review:
R4.1. Find an equation of the tangent line to the curve $y = e^x/x$ at the point $(1, e)$.
R4.2. Show that the function $y = Ae^{-x} + Bxe^{-x}$ satisfies the differential equation $y'' + 2y' + y = 0$.

HOMEWORK DAY 5 — Inverse trigonometric functions

Reading: §6.6.

Level 1: §6.6: 1,3,5,23,25,45.

Level 2: due Friday Sep 1
1. Graphs of inverse trig functions.
   (a) Sketch the graph of $y = \sin x, -\pi/2 \leq x \leq \pi/2$ and $y = \sin^{-1} x$ on the same screen.
   (b) Sketch the graph of $y = \tan x, -\pi/2 < x < \pi/2$ and $y = \tan^{-1} x$ on the same screen.
   (c) Sketch the graph of $y = 3 \tan^{-1}(x) + 2$
2. \( \text{§6.6: 11 (find \cos(\sin(x)))} \)

3. Find the derivatives of the given functions. \( \text{§6.6: 27,29,34} \)

4. \( \text{§6.6: 38 (implicit)} \)

5. \( \text{§6.6: 43,46 (limits)} \)

**Mixed Review:**

- R5.1. Evaluate the given integrals. \( \text{§6.2: 81,83,87} \)
- R5.2. Find the following limits. 
  
  \( \lim_{x \to \infty} (e^{-2x} \cos x) \),  
  
  \( \lim_{x \to (\pi/2)^+} e^{\tan x} \).

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**HOMEWORK DAY 6 — Inverse trigonometric functions, Hyperbolic functions**

**Reading:** \( \text{§6.7: pp 462-464 (skip inverse hyperbolic functions)} \)

**Level 1:** \( \text{§6.6: 61,63,65. \ §6.7: 1,3,9,31,32,33.} \)

**Level 2:** due Friday Sep 8

1. Evaluate the given integrals. \( \text{§6.6: 62,64,66} \)

2. Prove that \( \cosh^2(x) - \sinh^2(x) = 1 \)

3. Solve \( \cosh x = 2 \).

4. Use the definition \( \tanh(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}} \) to answer the questions below.
   
   (a) Is \( \tanh(x) \) odd or even? Show your work. (Use the definition of an odd/even function.)
   
   (b) Find all intercepts and all asymptotes of \( y = \tanh(x) \).
   
   (c) Show that \( f(x) = \tanh(x) \) is always increasing.
   
   (d) Use the information above to sketch a graph of \( y = \tanh(x) \).

5. If a water wave with length \( L \) moves with velocity \( v \) in a body of water with depth \( d \), then

\[
 v = \sqrt{\frac{gL}{2\pi}} \tanh \left( \frac{2\pi d}{L} \right)
\]

where \( g \) is acceleration due to gravity. Explain why the approximation

\[
 v \approx \sqrt{\frac{gL}{2\pi}}
\]

is appropriate in deep water.

6. \( \text{§6.7: 51.} \)

**Mixed Review:**

- R6.1. Consider the function \( f(t) = \frac{b}{1 + ae^{-kt}} \), where \( a, b, k \) are positive constants.
  
  (a) Sketch the graph of \( f \), clearly indicating coordinates of local extrema, inflection points, asymptotes, limiting behaviour near endpoints of domain. Clearly show all necessary work.
  
  (b) Use your result in a) to answer # 91 in Chapter 6 Review, page 483.

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**HOMEWORK DAY 7 — L’Hôpital’s Rule. Relative rates of growth**

**Reading:** \( \text{§6.8: pp 469–475} \)

**Level 1:** \( \text{§6.8: 1,2,3,4,7,8,11,12} \)

**Level 2:** due Friday Sep 8
1. Consider the drug response curve \( S(t) = At^2 e^{-kt} \), \( t \geq 0 \), where \( A, k > 0 \).
   (a) Find all intercepts and \( \lim_{t \to \infty} S(t) \).
   (b) Find the absolute minimum and maximum of \( S(t) \), if they exist. Justify your answer.
   (c) Use above to sketch a clearly labeled graph of \( S \).

2. Consider the function \( f(x) = x \ln x \).
   (a) State domain and intercepts of \( f \).
   (b) Find \( \lim_{x \to 0^+} f(x) \) and \( \lim_{x \to \infty} f(x) \).
   (c) Find the absolute minimum and maximum of \( f(x) \), if they exist. Justify your answer.
   (d) Use above to sketch a clearly labeled graph of \( f \).

3. Consider the function \( f(x) = \frac{\sin x}{x} \).
   (a) Show whether \( f \) is even or odd.
   (b) State domain and intercepts.
   (c) Find \( \lim_{x \to 0} f(x) \) and \( \lim_{x \to \infty} f(x) \).
   (d) Use above to sketch a clearly labeled graph of \( f \).

4. Find the following limits, if they exist. You must show all work.
   \[
   \begin{align*}
   (a) \quad & \lim_{x \to \infty} \frac{\ln(x)}{\sqrt{x}} \\
   (b) \quad & \lim_{x \to 1} \frac{\ln(x)}{\sqrt{x}} \\
   (c) \quad & \lim_{x \to 0^+} \frac{\ln(x)}{\sqrt{x}} \\
   (d) \quad & \lim_{x \to 0} \frac{x + 1 - e^x}{x^2} \\
   (e) \quad & \lim_{x \to \infty} \left( \ln(x) - \ln(x + 1) \right) \\
   (f) \quad & \lim_{n \to \infty} \left(1 + \frac{r}{n}\right)^n, r \in \mathbb{R}
   \end{align*}
   \]

5. Which one grows faster? Or do they grow at the same rate? Show your work.
   (a) \( \sqrt{x} \) or \( x^2 \)  
   (b) \( 4x^2 - 1 \) or \( -x^3 + 3x^2 + x - 1 \) (NEW)  
   (c) \( \ln x \) or \( \log_{10} x \)  
   (d) \( 2^x \) or \( e^x \)  
   (e) \( x^2 \) or \( x^2 + 1 \)  

Mixed Review:
   R7.1. Find the derivatives of
   \[
   (a) \quad f(x) = (\sqrt{x})^x 
   \]
   \[
   (b) \quad f(x) = \sinh(\ln x) 
   \]
   \[
   (c) \quad f(x) = \ln(\cosh 3x) 
   \]
   \[
   (d) \quad f(x) = \ln(xe^x) 
   \]
   \[
   (e) \quad f(x) = \frac{x^2 - 2}{3x^2 - 1} 
   \]

**Homework Day 8 — Integration by parts.**

**Reading:** §7.1: Introduction and examples 1-5.

**Level 1:** §7.1: 3,5,9,13,25.

**Level 2:** due Friday Sep 15

1. §7.1: 4,8,10,12
2. Find the following definite and indefinite integrals.
   \[
   \begin{align*}
   (a) \quad & \int x \sin(3x) \, dx \\
   (b) \quad & \int x \sin(3x^2) \, dx \\
   (c) \quad & \int \frac{\ln(x)}{x} \, dx \\
   (d) \quad & \int_0^t e^s \sin(t - s) \, ds
   \end{align*}
   \]

**Mixed Review:**

**Homework Day 9 — Trigonometric integrals.**

**Reading:** §7.2: Examples 1-6,9.

**Level 1:** §7.2: 1,3,7,21,23

**Level 2:** due Friday Sep 15

1. §7.2: 2,8,22
2. (a) Evaluate \( \int \sin(3x) \cos(5x) \, dx \) using the appropriate identities in §7.2, p500.
(b) Evaluate \( \int \sin(3x) \cos(5x) \, dx \) using integration by parts.
(c) Evaluate \( \int_0^{2\pi} \sin(3x) \cos(5x) \, dx \).

3. Find the following integrals

(a) \( \int \frac{\cos^5 \alpha}{\sin \alpha} \, d\alpha \)  
(b) \( \int \sin^5(2x) \cos(2x) \, dx \)  
(c) \( \int_0^{\pi/2} \cos^5(t) \, dt \)  
(d) \( \int \frac{\cos x + \sin 2x}{\sin x} \, dx \)

Mixed Review :
R9.1. For what \( c \) is \(-1/2\) the average value of \( (x - c) \sin(x) \) over the interval \([0, \pi/3]\)?

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**HOMEWORK DAY 10 — Trigonometric substitution.**

Reading: §7.3: Intro and examples 1-5.

Level 1: §7.3: 2,5,7

Level 2: due Friday Sep 15 §7.3: 6,8,10

Mixed Review :
R10.1. Evaluate the following integrals

(a) \( \int \ln(2x + 1) \, dx \)  
(b) \( \int \cos^2 x \, dx \)  
(c) \( \int \cos^3 x \, dx \)  
(d) \( \int x^2 \cos(mx) \, dx \)  
(e) \( \int x^5 \ln x \, dx \)  
(f) \( \int_0^{\pi/2} \sin^3 \theta \cos^2 \theta \, d\theta \)

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**HOMEWORK DAY 11 — Review for Exam.**

Further Practice:

§7.5: 2,3,6,8,14,16,20,30,57,63,76

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**HOMEWORK DAY 13 — Partial Fractions.**

Level 1: §7.4: 1,11

Level 2: due Friday Sep 22 §7.4: 7, 15, 16, 22, 51

Mixed Review :
R13.1. Evaluate the following integrals

(a) \( \int_1^2 \ln x \, dx \)  
(b) \( \int e^{3x} \, dx \)

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**HOMEWORK DAY 14 — Partial Fractions.**

Level 1: §7.4: 5

Level 2: due Friday Sep 29

1. §7.4: 13, 21, 24, 54

2. Evaluate \( \int \frac{dx}{x^2 + k} \) by considering several cases for the constant \( k \).

Mixed Review :
R14.1. Evaluate the following integrals

(a) \( \int_1^2 \frac{x}{(x+1)^2} \, dx \)  
(b) \( \int_1^2 \frac{\sqrt{x^2 - 1}}{x} \, dx \)  
(c) \( \int \tan^2 \theta \sec^4 \theta \, d\theta \)

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**HOMEWORK DAY 15 — Numerical Integration.**
Level 1: §7.7: 1, 11

Level 2: due Friday Sep 29 §7.7: 2, 5, 6, 16

Mixed Review:

R15.1. Evaluate the following integrals
   
   (a) \[ \int_{1}^{2} \frac{dx}{x^2 + x} \]
   
   (b) \[ \int_{0}^{\pi/6} t \sin(2t) \, dt \]
   
   (c) \[ \int_{2}^{e} \frac{dx}{x \ln x} \]

R15.2. Let \( f(x) = x^2 e^{-x} \).
   
   (a) Find domain, intercepts, intervals where \( f \) is increasing and decreasing, limits as \( x \to \infty, x \to -\infty \).
   
   (b) Use the above information to sketch graph of \( f \).

HOMEWORK DAY 16 — Numerical Integration.

Level 1: none

Level 2: due Friday Sep 29

1. §7.7: 30
2. §7.7: 20

3. How large should \( n \) be to guarantee that the error in the numerical approximation of \( \int_{0}^{1} e^{x^2} \, dx \) is less than \( 10^{-5} \) if you use
   
   (a) the Trapezoid rule?
   
   (b) Simpson’s rule?
   
   Make sure to clearly explain your answers.

Mixed Review:

R16.1. Let \( f(x) = x \ln x \).
   
   (a) Find domain, intercepts, intervals where \( f(x) > 0 \), limits as \( x \to \infty \), \( x \to 0^+ \).
   
   (b) Use the above information (and no more) to sketch graph of \( f \).
   
   (c) Find all local extrema of \( f \) and add them to your graph.

HOMEWORK DAY 17 — Improper Integrals.

Level 1: §7.8: 7, 9, 23, 27, 32.

Level 2: due Friday Oct 6 §7.8: 14, 16, 24, 25 34, 40, 62 (Suggestion: let \( a = M/(2RT) \))

Mixed Review:

R17.1. Evaluate the following integrals
   
   (a) \[ \int \frac{dx}{x^2 \sqrt{x^2 + 4}} \]
   
   (b) \[ \int e^x \cos x \, dx \]

R17.2. Show that \( \cosh^2 x - \sinh^2 x = 1 \).

HOMEWORK DAY 18 — First order differential equations.

Level 1: §9.1: 5, 11.

Level 2: due Friday Oct 6

1. §9.1: 2, 3, 4, 7, 9, 10, 12, 14 (denote the constant of proportionality by \( k \), where \( k > 0 \))

2. Find the solution to the initial value problem
   
   \[ \frac{dy}{dx} = \frac{1}{4 + x^2}, \quad y(2) = 1 \]
3. Write the solution to the initial value problem

\[ \frac{dy}{dx} = e^{-x^2}, \quad y(5) = 0 \]

in integral form.

**Mixed Review:**

R18.1. Let \( T_n \) be the approximation of the integral \( I = \int_a^b f(x) \, dx \) by the trapezoid rule, using \( n + 1 \) uniformly spaced points \( x_j = a + j\Delta x, \quad j = 0, \ldots, n \), where \( \Delta x = (b - a)/n \).

(a) Write down a formula for \( T_n \).

(b) Find the approximation \( T_4 \) of the integral \( I = \int_0^1 e^x \, dx \).

(c) Sketch a graph of \( f(x) = e^x \) and the area given by the approximation \( T_4 \) (b). Using solely your sketch, determine whether \( T_4 \) is an over- or an underestimate. Explain.

(d) For any function \( f \) with a continuous second derivative, the error of the trapezoid approximation can be shown to be

\[ |I - T_n| = \frac{(b - a)^3}{12n^2} |f''(c)| \]

for some \( c \in [a, b] \). Use this formula to find an upper bound for the error in your approximation in (b). Compare this upper bound with the actual error.

(e) What should the value of \( n \) be so that the approximation error \( |I - T_n| \) of the integral in (b) is guaranteed to be less than \( 10^{-4} \)?

**HOMEWORK DAY 19 — Direction fields, solution curves, autonomous systems.**

**Level 1:** §9.2: 1,7,9.

**Level 2:** due Friday Oct 6

1. §9.2: 2, 3-6

2. For the differential equation you proposed in §9.1: 14,

(a) Draw the direction field for the differential equation.

(b) For which initial coffee cup temperature does the temperature remain constant? State the steady solution.

(c) In your plot in (a), sketch a few sample solution curves, including all steady solutions, as well as solutions with \( T(0) < 20^\circ C \), and \( T(0) > 20^\circ C \).

(d) For any solution \( T(t) \), use your graphs in (c) to determine \( \lim_{t \to \infty} T(t) \). Interpret your result physically.

3. Draw the directions fields and a few solution curves for the following differential equations

(a) \( \frac{dy}{dx} = y \)  
(b) \( \frac{dy}{dx} = x \)  
(c) \( \frac{dy}{dx} = x^2 \)

(d) \( \frac{dy}{dx} = (y - 1)(y - 2) \)  
(e) \( \frac{dy}{dt} = k(y - a) \)  
(f) \( \frac{dP}{dt} = kP(1 - \frac{P}{P_0}) \)

where all parameters \( k, a, P_0 \) are positive constants. Which of these differential equations can you solve using methods of Calculus I?

**Mixed Review:**

R19.1. For which values of \( p \) does \( \int_a^\infty \frac{dx}{x^p}, \quad a > 0 \) converge? For which does it diverge?
R19.2. Find the value of the following improper integrals or determine that they diverge. Use correct notation throughout.

(a) \( \int_{-\infty}^{\infty} \cos(\pi t) \, dt \)

(b) \( \int_{0}^{1} \frac{x - 1}{\sqrt{x}} \, dx \)

(c) \( \int_{0}^{3} \ln(x) \, dx \)

(d) \( \int_{-\infty}^{\infty} \frac{x^2}{9 + x^6} \, dx \)

R19.3. Let \( f(x) = \frac{\ln x}{x^2} \).

(a) Find domain, intercepts, values of \( x \) where \( y > 0 \), limits as \( x \to \infty \), \( x \to 0^+ \).

(b) Use the above information (and no more) to sketch graph of \( f \).

(c) Find all local extrema of \( f \) and add them to your graph.

HOMEWORK DAY 20 — Separation of variables.

Level 1: §9.3: 3, 11, 13, 16.

Level 2: due Wednesday Oct 11

1. §9.3: 10, 18, 20

2. Solve the differential equation you proposed in §9.1: 14, with initial temperatures

(a) \( T(0) = T_0 < 20^\circ C \)  (b) \( T(0) = T_0 > 20^\circ C \)  (c) \( T(0) = T_0 = 20^\circ C \)

In each case, use your solution to determine \( \lim_{t \to \infty} T(t) \).

Compare with your graphs from DAY 18, Problem 2.

3. A function \( y(t) \) satisfies the differential equation \( y' = y^2 \).

(a) What are the steady solutions?

(b) Sketch the direction field and three solution curves, corresponding to three initial conditions \( y(0) > 0 \), \( y(0) = 0 \), \( y(0) < 0 \).

(c) Find the solutions \( y(t) \) if

(i) \( y(0) = 1 \)

(ii) \( y(0) = 0 \)

(iii) \( y(0) = -1 \)

and compare with the sketch drawn in part (e). What happens to each of the three solutions as time increases?

4. Consider the differential equation

\[ \frac{dy}{dx} = xy^3 \]

(a) Find the solution \( y(x) \) if \( y(0) = 2 \) and show that the solution becomes unbounded (approaches \( \infty \)) as \( x \) approaches a finite value \( x_c \). That is, the solution curve has a vertical asymptote at \( x = x_c \). Find \( x_c \).

(b) Repeat for the initial condition \( y(0) = -1 \).

Mixed Review:

R20.1. Given the linear, homogeneous differential equation \( y'' + 2y' + 2y = 0 \) where \( y = y(t) \). Show that \( y_1(t) = e^{rt} \sin t \) and \( y_2(t) = e^{rt} \cos t \) both solve the equation for a particular value of \( r \). Find that value.

R20.2. §7.7: 22

R20.3. §7.8: 22.

HOMEWORK DAY 21 — Miscellaneous examples.

Level 2: due Wednesday Oct 11
3. Consider the differential equation
\[ \frac{dP}{dt} = 0.1P(1 - \frac{P}{2000}) \]
(a) Sketch the direction field and some solution curves. What are the steady solutions?
(b) Solve the equation if \( P(0) = 100 \).
(c) Solve the equation if \( P(0) = 2000 \).
(d) Solve the equation if \( P(0) = 4000 \).
In each of (b-d), find the limit as \( t \to \infty \). Does your result agree with the solution curves plotted in (a)?

Mixed Review:
R21.1. Show that \( y(x) = e^{-x^2} \int_0^x e^{t^2} \, dt \), which is known as Dawson’s integral, is the solution of the initial value problem \( y'(x) = 1 - 2xy \), \( y(0) = 0 \).
R21.2. §7.8: 18, 28.
R21.3. §9.3: 16.

HOMEWORK DAY 22 — Euler’s method.

Level 1: none

Level 2: due Wednesday Oct 18
1. §9.2: 20
2. Use Euler’s method with step size 0.5 to approximate \( y(3) \), where \( y(t) \) solves the initial value problem
\[ \frac{dy}{dt} = y - 2t \quad , \quad y(1) = 2 \]
3. Consider the initial value problem
\[ \frac{dP}{dt} = P(2 - P) \quad , \quad P(0) = 1/2 \]
(a) Sketch the direction field and some solution curves. Clearly indicate the one solution that solves the initial value problem.
(b) Find the solution \( P(t) \) of the initial value problem. What is the limit as \( t \to \infty \)? Does your result agree with the solution curve plotted in (a)?
(c) Approximate \( P(1) \) using Euler’s method with \( \Delta t = 1/2 \). Compare the approximation with the your exact result in (b).
(d) Repeat (c) using \( \Delta t = 1/4 \).

Mixed Review:
R22.1. Psychologists interested in learning theory study learning curves. A learning curve is the graph of a function \( P(t) \), the performance of someone learning a skill as a function of the training time \( t \). A model for learning is given by
\[ \frac{dP}{dt} = k(M - P) \]
where \( M \) is the maximum level of performance of which the learner is capable.
(a) Draw a direction field and several solution curves for this differential equation.
(b) At what level of performance is the rate of improvement $dP/dt$ the largest?

(c) Find the performance $P(t)$ if the initial performance is $P(0) = M$.

(d) Find the performance $P(t)$ if the initial performance is $P(0) = P_0 < M$.

(e) What does the number $k$ measure?

R22.2. §7.8: 31

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**HOMEWORK DAY 23 — Sequences.**

**Level 1:** §11.1: 1,5,13,23,25,27,29,35,39,41,43.

**Level 2:** due Wednesday Oct 18 §11.1: 9,14,24,26,28,38,64,65,67.

**Mixed Review :**

R23.1. A glucose solution is administered intravenously into the bloodstream at a constant rate $r$. As the glucose is added, it is converted into other substances and removed from the bloodstream at a rate that is proportional to the concentration at that time. Thus a model for the concentration $C = C(t)$ of the glucose solution in the bloodstream is

$$\frac{dC}{dt} = r - kC$$

where $r$ and $k$ are positive constants. Assume that $C$ is measured in $mg/mL$ and $t$ in minutes.

(a) Sketch the direction field and a few solution curves.

(b) Assume $k$ is known. At what rate should the solution be administered so that the glucose concentration approaches a desired level of .9 mg/mL?

(c) Find the solution to the differential equation if $C(0) = r/k$.

(d) Find the solution to the differential equation if $C(0) = C_0 > r/k$. (Since the initial concentration is bigger than $r/k$ you may assume that $C(t) > r/k$ at all times, in view of your direction field.)

R23.2. §7.8: 22.

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**HOMEWORK DAY 26 — Series: definition, convergence, divergence theorem.**

**Examples (geometric).**

**Level 1:** §11.2: 1,5,17,21.

**Level 2:** due Friday Oct 27

1. §11.2: 3,6,15,16,18
2. (a) What does it mean to state that $\sum_{k=1}^{\infty} a_k = L$?
   (b) Assume $\sum_{k=1}^{\infty} a_k = L$. What is $\lim_{k \to \infty} a_k$? What is $\lim_{n \to \infty} s_n$, where $s_n = \sum_{k=1}^{n} a_k$?
   (c) Find the value of the series $\sum_{k=1}^{\infty} a_k$ if its partial sums are given by $s_n = \frac{n^2 - 1}{4n^2 + 1}$.
3. Fill in the blanks using either *may* or *must*.
   (a) A series with summands tending to 0 ________ converge.
   (b) A series that converges ________ have summands that tend to zero.
   (c) If a series diverges, then the summands ________ not tend to 0.
   (d) If a series diverges, then the Divergence Test ________ succeed in proving the divergence.
(e) If $\sum_{n=10}^{\infty} a_n$ diverges, then $\sum_{n=1000}^{\infty} a_n$ diverge.

4. For the series $\sum_{n=1}^{\infty} 1$, find a formula for the partial sums $s_N$, the limits $\lim_{n \to \infty} a_n$ and $\lim_{N \to \infty} s_N$, and determine whether the series converges or diverges.

5. For each of the following series: compute the partial sums $S_N = \sum_{n=1}^{N} a_n$ for $N = 1, \ldots, 10$, graph both the sequence $\{a_n\}$ and the sequence of partial sums $\{S_N\}$, and determine, as best you can, whether the series converges or diverges.

(a) $\sum_{n=1}^{\infty} 1$

(b) $\sum_{k=1}^{\infty} \frac{(-1)^k}{k}$

6. Evaluate the following series or determine that they diverge (with very brief explanation).

(a) $\sum_{n=0}^{\infty} \left(\frac{2}{3}\right)^n$

(b) $\sum_{n=-3}^{\infty} \frac{1}{(-5)^n}$

(c) $1 + 0.4 + 0.16 + 0.064 + \ldots$

(d) $\sum_{j=1}^{\infty} \cos(\pi j)$

(e) $\sum_{l=1}^{\infty} \sin(2\pi l)$

(f) $\sum_{n=4}^{\infty} 5^{-n}3^{-n}4^n$

Mixed Review:

R26.1. §7.8: 21

R26.2. §7.8, # 62 (average speed of molecules in ideal gas) Hint: begin by setting $a = M/(2RT)$, thus simplifying the integrand.

HOMEWORK DAY 27 — Series: more examples (harmonic, alternating harmonic, telescoping), basic properties, tail end of a series.

Level 1: §11.2: 29,35,39,43.

Level 2: due Friday Oct 27

1. §11.2: 32,36,38,40,44

2. Evaluate the following series or determine that they diverge (with very brief explanation).

(a) $\sum_{n=3}^{\infty} (5^{-n} + 2 \cdot 3^{-n})$

(b) $\sum_{n=1}^{\infty} \frac{7^{n-1}}{(-9)^n}$

(c) $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$

(d) $\sum_{n=1}^{\infty} (\tan^{-1}(n + 2) - \tan^{-1} n)$

3. Using only the formula

$$A = \sum_{n=0}^{\infty} r^n = \frac{1}{1-r} \text{ if } |r| < 1,$$

find the value of $B = \sum_{n=2}^{\infty} \frac{1}{n^2}$ in two different ways:

(a) factor out the leading term of $B$ to reduce to a constant times a term in form $A$.

(b) look at the difference between $A$ and $B$ and then use $A$ to find $B$.

Mixed Review:

R27.1. Sketch the graphs of $\sin(x), \cos(x)$. (You need to know these by heart.) Using these graphs and the definitions of $\tan(x), \sec(x), \sin^{-1}(x), \cos^{-1}(x), \tan^{-1}(x)$, obtain the graphs of these other trig and inverse trig functions. All graphs should clearly show domains, intercepts, limiting behaviour.
R27.2. Find the derivatives of the following functions

(a) \( f(x) = x^2 \)  
(b) \( f(x) = \tan^{-1} \frac{x}{x} \)  
(c) \( f(x) = \sinh(\ln x) \)  
(d) \( f(x) = \int_1^{\sqrt{x}} \tan^{-1} s \, ds \)  
(e) \( f(x) = x \sin^{-1}(x^3) \)

R27.3. Solve the initial value problem

\[ \frac{dy}{dt} = \frac{y^2}{1 + t^2}, \quad y(1) = -\frac{1}{2}. \]

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**HOMEWORK DAY 28 — Integral Test, p-series.**

**Level 1:** §11.3: 3,7,11,17,21.

**Level 2:** due Friday Nov 3

1. §11.3: 4,12,22

2. Determine whether the following series converge or diverge (with a brief, but complete, explanation).

   (a) \( \sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} \)  
   (b) \( \sum_{n=1}^{\infty} \frac{1}{n^2 + 1} \)  
   (c) \( \sum_{n=1}^{\infty} \frac{1}{n^5} \)

   (d) \( 1 + \frac{10}{4} + \frac{10}{9} + \frac{10}{16} + \ldots \)  
   (e) \( \sum_{n=1}^{\infty} \tan^{-1} n \)  
   (f) \( \sum_{n=100}^{\infty} \frac{4^n}{3^n + 3} \)

   (g) \( \sum_{n=2}^{\infty} 2^n \)  
   (h) \( \sum_{n=1}^{\infty} e^{-n} \)

3. (a) Write the number 0.2 = 0.22222... as a geometric series. Then evaluate that series to express the number as a ratio of integers.
   (b) Use series to show that 0.9 = 1.0 .

4. Find the values of \( x \) for which the series \( \sum_{n=1}^{\infty} \frac{x^n}{n} \) converges. Find the sum of the series for those values of \( x \). (Note: this is the geometric series with \( r = x/3 \) starting at \( n = 1 \).)

5. Find the value of \( r \) if \( \sum_{n=2}^{\infty} r^n = 2 \)

**Mixed Review :**
R28.1. State the definitions of \( \sinh(x), \cosh(x), \tanh(x) \), and use them to obtain their graphs. All graphs should clearly show domains, intercepts, limiting behaviour.

R28.1. Evaluate the following indefinite integrals.

   (a) \( \int \frac{1 + x - x^2}{x^2} \, dx \)  
   (b) \( \int t \sin(2t) \, dt \)  
   (c) \( \int \frac{x^3}{1 + x^4} \, dx \)

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**HOMEWORK DAY 29 — Comparison Tests**

**Level 1:** §11.4: 1,3,13.

**Level 2:** due Friday Nov 3 §11.4: 2,4,6,10,14,16,18,24

**Mixed Review :**
R29.1. Sketch the graphs of the following functions: \( \ln(x), e^x, 2^x, (1/2)^x \).
R29.2. Evaluate the following definite and indefinite integrals.

(a) \( \int \frac{x^3}{1-x^2} \, dx \)  

(b) \( \int_0^1 \sqrt{1-x^2} \, dx \)  

(c) \( \int_0^1 ve^v \, dv \)  

HOMEWORK DAY 30 — Alternating series

Level 1: §11.5: 1,7,19

Level 2: due Friday Nov 3

§11.5: 2,4,6,8,17,20,32 (explain your answers.)

Mixed Review:

R30.1. (a) What does it mean for a series to converge? What is the value of a series?

(b) A series \( \sum_{n=n_0}^{\infty} a_n \), with \( a_n > 0 \), converges if the summands \( a_n \) approach zero sufficiently fast! How fast is fast enough?

R30.2. Evaluate the following definite and indefinite integrals.

(a) \( \int_0^t (t-s)^2 \, ds \)  

(b) \( \int \tanh x \, \text{sech}^2 x \, dx \)  

(c) \( \int e^\theta \sin \theta \, d\theta \)  

HOMEWORK DAY 31 — Absolute convergence. Ratio test.

Level 1: §11.6: 1,3,15.

Level 2: due Friday Nov 10

1. §11.6: 7,8,9,12

2. Determine whether the following series converge or diverge, using any of the tests we have learnt about. If they converge, do they converge absolutely or conditionally? In each case, give a concise answer stating the test that you used and an explanation. For example:

For problem 11.6:12 in the online hw you could write “Conv abs by direct comparison test, since \( |a_n| = |\sin(4n)/4^n| \leq 1/4^n \), and \( \sum 1/4^n \) is a converging p-series, so \( \sum |a_n| \) converges”)

For problem 11.6:9 in the online hw you could write “diverges by divergence test since the exponential \( 1.1^n \) grows faster than the algebraic \( n^4 \), so \( |a_n| \) does not approach 0.”)

(a) \( \sum_{k=2}^{\infty} \frac{1}{\sqrt{k} + 4} \)  

(b) \( \sum_{n=2}^{\infty} \frac{1}{n(lnn)^2} \)  

(c) \( \sum_{n=3}^{\infty} \frac{4^n}{5^n - 1} \)  

(d) \( \sum_{n=1}^{\infty} (-1)^n \frac{3n-1}{2n+1} \)  

(e) \( \sum_{n=1}^{\infty} (-1)^n \frac{n^2}{n^3+1} \)  

(f) \( \sum_{n=1}^{\infty} \frac{100^n}{n!} \)  

(g) \( \sum_{n=1}^{\infty} \frac{\cos(n)}{n^2} \)  

3. Which grows faster as \( n \to \infty \), the exponential function \( 100^n \) or the factorial function \( n! \). Explain. Hint: use your result from problem 1(f) above.

4. Explain why, if \( \sum a_n \) converges, then \( \sum \frac{1}{a_n} \) diverges.

5. §11.5, # 23 (approximate an alternating series to within a prescribed error)
Mixed Review:
R31.2. Evaluate the following definite and indefinite integrals. Some of the definite ones may be improper, careful.

(a) \(\int_{100}^{\infty} \frac{1}{\sqrt{x}} \, dx\)  
(b) \(\int_{-\infty}^{\infty} e^{-x} \, dx\)  
(c) \(\int_{-1}^{1} \frac{1}{x} \, dx\)  
(d) \(\int_{e}^{e^2} \frac{\ln(\ln x)}{x} \, dx\)

R31.3. Chapter 6 Review (p 482): 21, 27, 41, 42 (derivatives)

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HOMEWORK DAY 32 — Power series

Level 1: §11.8: 3, 7, 11.
Level 2: due Friday Nov 10

1. §11.8: 16, 19
2. Which of the following are power series?

(a) \(\sum_{n=0}^{\infty} (3x)^n\)  
(b) \(\sum_{n=0}^{\infty} \sqrt{x^n}\)  
(c) \(\sum_{n=0}^{\infty} (x + 2)^{2n}\)

3. Determine the radius and the interval of convergence of the power series

\(\sum_{n=1}^{\infty} \frac{3^n (x + 4)^n}{\sqrt{n}}\)

Mixed Review:

R32.1. Review Chapter 11 (pp 802-804), Concept Check: 1, 3, 4, 6.
R32.2. Find the derivatives of the following functions.

(a) \(f(x) = \sin(x^3)\)  
(b) \(f(x) = \cosh(x)\)  
(c) \(f(t) = \frac{1}{1 + 2t^2}\)

(d) \(f(x) = xe^x\)  
(e) \(F(x) = \int_{0}^{x} \frac{\sin t}{t} \, dt\)  
(f) \(g(t) = \frac{1}{(1 + t)^2}\)

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HOMEWORK DAY 33 — Representing functions as power series

Level 1: §11.9: 1, 2, 3, 5.
Level 2: due Friday Nov 10

1. §11.9: 8, 15
2. Use the geometric series to find a power series representation centered at \(x = 0\) and its radius of convergence, for

(a) \(f(x) = \frac{1}{4 + 2x^2}\)  
(b) \(f(x) = \tan^{-1}(2x)\)

3. (a) Find a power series representation of \(f(x) = \frac{1}{8 + x}\). Find its radius of convergence.

(b) Use differentiation to find a power series representation of \(f(x) = \frac{1}{(8 + x)^2}\). State its radius of convergence.
(c) Find a power series representation of \( f(x) = \frac{1}{(8 + x)^3} \). State its radius of convergence.

(d) Find a power series representation of \( f(x) = \frac{x^2}{(8 + x)^3} \). State its radius of convergence.

4. Find a power series representation of \( \frac{1}{(D+a)^2} \) in terms of the variable \( d/D \), about \( d/D = 0 \).

Mixed Review:

R33.1. Review Chapter 11 (pp 802-804), True-False: 1,4,5,6,8,9,10,11,12,17,18,19.

R33.2. Evaluate the following definite and indefinite integrals. Some of the definite ones may be improper, careful.

\[
\begin{align*}
(a) & \quad \int_0^1 \frac{x}{\sqrt{1-x}} \, dx \\
(b) & \quad \int_0^1 \frac{1+x}{1-x} \, dx \\
(c) & \quad \int_0^1 \frac{1+2x}{1-x^2} \, dx \\
(d) & \quad \int \frac{dx}{\sqrt{8-2x-x^2}} \\
(e) & \quad \int \frac{16\cos(x)}{\sin^2(x)-5\sin(x)+6} \, dx \\
(f) & \quad \int_2^\infty x^2 \, dx
\end{align*}
\]

HOMEWORK DAY 34 — Taylor series. Derivation

Level 1: §11.10: 3,5,13.

Level 2: due Friday Nov 17

1. §11.10: 6,16 What is its radius of convergence?

2. Find the Taylor series for \( f \) about \( x = 4 \) if

\[
f^{(n)}(4) = \frac{(-1)^n n!}{3^n (n+1)}
\]

What is its radius of convergence?

3. Find the Maclaurin series for \( f(x) = \cosh(x) \).

4. Find the Taylor series for \( f(x) = \sqrt{x} \) centered at \( x = 4 \).

5. Find the Taylor series for \( f(x) = x - x^3 \) about \( x = 2 \).

6. If \( f(x) = 2x - x^2 + \frac{1}{3}x^3 - \ldots \) converges for all \( x \), what is \( f''''(0) \)? Can you find it without differentiating \( f(x) \)?

Mixed Review:

R34.1. Determine whether the following series converge conditionally, converge absolutely, or diverge. Chapter 11 Review (p 803): 11,12,13,14.


R34.3. Solve the following differential equations

\[
\begin{align*}
(a) & \quad \frac{dy}{dt} = 1 - t \\
(b) & \quad \frac{dy}{dt} = 1 - t + x - tx \\
(c) & \quad 2ye^{y^2} y' = 2x + 3\sqrt{x}
\end{align*}
\]

HOMEWORK DAY 35 — Obtaining new Taylor series from old

Level 1: §11.10: 30,31.

Level 2: due Friday Nov 17

Find the first 5 nonzero terms of the Maclaurin series in the following problems. For # 38 and 48, also find a formula for the nth term and write out the Maclaurin series using summation notation.
Mixed Review:
R35.1. Determine whether the following series converge conditionally, converge absolutely, or diverge. Review Chapter 11 (p 803), 15,16,20,24.
R35.2. Find the sum of the series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^5}$ correct to within $5 \cdot 10^{-5}$.


Level 1: §11.11: 3,4,5,10.
Level 2: due Friday Nov 17
1. (a) Write down a formula for the Taylor series of $f$ about $x = a$.
   (b) Write down a formula for the Taylor polynomial $p_n$ of degree $n$ of a function $f(x)$ about a basepoint $x = a$?
   (c) If $p_n$ is used to approximate a function $f$ about $x = a$, Write down an upper bound for the error $|f(x) - p_n(x)|$.
   (d) Find $p_2(x)$ for $f(x) = e^x$ about $x = 0$. Write down a formula for an upper bound for $|f(x) - p_2(x)|$. Can you find an upper bound for $|f(x) - p_2(x)|$ if $|x| < .1$?

2. Find the first 5 nonzero terms of the Taylor series of $f(x) = \sin x$ at $a = \pi/6$. State the linear approximation of $f$ about $a = \pi/6$. State the Taylor polynomial of degree 2 $p_2$ for $f$ about $a = \pi/6$. How large is the magnitude of the error in the approximation $f(x) \approx p_2(x)$, for $x \in [0, \pi/3]$, at most?

3. Here we address: How good is the approximation $\sin \theta \approx \theta$ if $\theta$ is small?
   (a) Find the Taylor series for $f(\theta) = \sin \theta$ about $\theta = 0$. Find the linear approximation $p_1(\theta)$.
   (b) State the Alternating Series Remainder Theorem.
   (c) Find an upper bound for $|f(\theta) - p_1(\theta)|$ if $|\theta| \leq 0.1$ using the Alternating Series Remainder Theorem. Explain why you can use the ASRT.
   (d) Find an upper bound for $|f(\theta) - p_1(\theta)|$ if $|\theta| \leq 0.1$ using Taylor’s Inequality. Which is a tighter bound, your result in (c) or in (d)?

4. Find the Maclaurin series for the following functions and state their radius and interval of convergence.
   (a) $f(x) = \sin(x^3)$
   (b) $f(t) = \frac{1}{1 + 2t^2}$
   (c) $f(x) = xe^x$
   (d) $F(x) = \int_0^x \frac{\sin t}{t} dt.$
   (e) $g(t) = \frac{1}{(1 + t)^2}$

Mixed Review:
R36.1. Review Chapter 11 (pp 802-804), Concept Check: 9,11.
R36.3. Let $f(x) = \sum_{n=1}^{\infty} \frac{x^n}{n^2}$. Find series representations for $f'$, $f''$. Find the intervals of convergence of $f$, $f'$, and $f''$. (Remember: the radius of convergence does not change under differentiation or integration! So, you only have to find it for one of these series. However, the interval of convergence can change.)
R36.4. Determine whether the following series converge absolutely, conditionally, or diverge. If it converges, can you find its value?

(a) \[ \sum_{n=0}^{\infty} \frac{1 + n^3}{1 + 2n^3} \]

(b) \[ \sum_{n=10}^{\infty} \frac{(-1)^n (1 + n)}{1 + 2n^3} \]

(c) \[ \sum_{n=0}^{\infty} \frac{2(-1)^n n^{n+1}}{5^n} \]

(d) \[ \sum_{n=1}^{\infty} \frac{(-1)^n \sqrt{n}}{n + 1} \]

(e) \[ \sum_{n=0}^{\infty} \frac{(-1)^n 3^n}{n!} \]

(f) \[ \sum_{n=1}^{\infty} \frac{(-1)^n 2^n}{n!} \]

**Homework Day 39 — Applications of Taylor series to physics and Engineering. Other Series: Fourier series.**

**Level 1:** none

**Level 2:** due Friday Dec 1

1. A simple pendulum. An idealized simple pendulum is given by a mass \( m \) hanging from a massless string of length \( L \). Its motion is described by the angle \( \theta(t) \), where \( t \) is time. The distance travelled by the pendulum is the arclength \( s(t) = L\theta(t) \). According to Newton’s 2nd law, the pendulum mass \( \times \) acceleration equals the restoring force \( F_{net} \) acting on it.

\[ mL\theta''(t) = F_{net} \]

where \( F_{net} = mg\sin\theta \) (see picture). For small angles one often uses the approximation \( \sin\theta \approx \theta \) to replace this differential equation by the simpler equation

\[ mL\theta''(t) = mg\theta \]

(which is easy to solve, as you’ll find out in Math 316). Question: If the angle swings with \( -\pi/10 \leq \theta \leq \pi/10 \), what is an upper bound for the error made in the approximation \( \sin\theta \approx \theta \) ?

2. (§12.11, #33) An electric dipole consists of two electric charges of equal magnitude and opposite signs. If the charges are \( q \) and \( -q \) and are located at a distance \( d \) from each other, then the electric field \( E \) at the point \( P \) in the figure is

\[ E = \frac{q}{D^2} - \frac{q}{(D + d)^2} \]

By expanding this expression for \( E \) as a series in powers of \( d/D \), show that \( E \) is approximately proportional to \( 1/D^3 \) when \( P \) is far away from the dipole (that is, when \( D \) is much bigger than \( d \), so that \( d/D \ll 1 \)).

3. When a voltage \( V \) is applied to a series circuit consisting of a resistor \( R \) and an inductor \( L \), the current at time \( t \) is \( I = \left( \frac{V}{R} \right) \left( 1 - e^{-Rt/L} \right) \). Use Taylor series to deduce that \( I \approx Vt/L \) if \( R \) is small.
4. What does the statement \( f(x) = \sum_{n=1}^{\infty} \frac{1}{n} \sin(nx) \) mean? Can you write down a finite sum approximation of \( f(x) \)? The series on the right hand side is called a Sine Series.

**Mixed Review:**

R39.1. Show that \( \cosh x > 1 + \frac{1}{2}x^2 \) for all \( x \). (Hint: use Taylor series!)

R39.2. Sketch the graphs of the following functions, clearly indicating domains, limiting behaviour, intercepts: \( \sin^{-1}(x), \cos^{-1}(x), \tan^{-1}(x), \sinh(x), \cosh(x), \tanh(x), \ln(x), e^x \).

R39.3. Find the derivatives of the following functions
(a) \( f(x) = x^{2^x} \)  
(b) \( f(x) = \frac{\tan^{-1}x}{x} \)  
(c) \( f(x) = \sinh(\ln x) \)  
(d) \( f(x) = \int_1^x \tan^{-1}s \, ds \)

R39.4. Evaluate the following definite and indefinite integrals.
(a) \( \int \frac{1+x-x^2}{x^2} \, dx \)  
(b) \( \int \frac{x^3}{1+x^4} \, dx \)  
(c) \( \int \frac{x^3}{1-x^2} \, dx \)

(d) \( \int_0^1 \sqrt{1-x^2} \, dx \)  
(e) \( \int_0^1 (t-s)^2 \, ds \)  
(f) \( \int \tanh x \sech^2 x \, dx \)

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**HOMEWORK DAY 40**

*Applications to complex numbers.*

**Level 1:** Appendix H 1,3,5,7,11,13,15,17

**Level 2:** due Friday Dec 1

1. Appendix H: 2,6,8,10,12,16

2. Find all solutions to (a) \( 2x^2 + 2x = 5 \), (b) \( 2x^2 + 5 = 2x \) and plot them in the complex plane.

3. Simplify (a) \((3+5i)(2i-1)\), (b) \((3+5i)/(2i-1)\).

4. Find the polar coordinates \( r \) and \( \theta \) (in radians) of the following points in the complex plane:
   (a) \(-4i\)  
   (b) \(12-5i\)  
   (c) \(1/(1+i)\)  
   (d) \((1-2i)(8-3i)\)

5. (a) Sketch the set of all complex numbers in the complex plane with \( r = 5 \).
   (b) Sketch the set of all complex numbers in the complex plane with \( \theta = \pi/3 \).

**Mixed Review:**

R40.1. Chapter 11 Review (p 804), # 47,48,50,51. (Find the Maclaurin series for \( f \) and its radius of convergence.)

R40.2. §9.1, #2.

R40.3. (a) Write down the linear approximation of a function \( f(x) \) at \( x = a \).
   (b) Find the linear approximation for \( f(x) = \sqrt{1+x} \) at \( a = 0 \). Use it to approximate \( \sqrt{1.1}, \sqrt{0.9} \).
   (c) Find the linear approximation for \( f(x) = \sqrt{x} \) at \( a = 1 \).

R40.4. For the function \( f(x) = (x-2)e^{-x} \)
   (a) State its domain and all intercepts.
   (b) Find \( \lim_{x \to \infty} f(x) \) and \( \lim_{x \to -\infty} f(x) \).
   (c) Find all points of zero slope and all inflection points.
   (d) Sketch a graph of the function.
   (e) State its absolute maximum value.

R40.5. Review Chapter 6 (p 484), # 121
R40.6. Find the following integrals
(a) \( \int_{-2}^{1} \sqrt{4-x^2} \, dx \)
(b) \( \int t^5 \sqrt{t^2 + 2} \, dt \)
(c) \( \int \frac{dt}{1-t^2} \)

\[ \text{HOMEWORK DAY 41} \]

**Complex numbers.**

**Level 1:** Appendix H 19,20,21,42,43,44,45,46

**Level 2:** due Friday Dec 1

1. Find the real and imaginary parts of the following numbers and plot them in the complex plane.
   (a) \( z = e^{2-\pi i/4} \)  
   (b) \( z = e^{-1-i} \)

2. By writing the individual factors on the left in exponential form, as \( re^{i\theta} \), performing the needed operations, and finally changing back to Cartesian coordinates, show that
   (a) \( i(1 - \sqrt{3}i)(\sqrt{3} + i) = 2(1 + \sqrt{3}i) \)  
   (b) \( (-1 + i)^7 = -8(1 + i) \)

3. Write the following numbers in the form \( re^{i\theta} \) and in the form \( a + ib \):
   (a) \( (1 + i)^{20} \)  
   (b) \( (1 - \sqrt{3}i)^5 \)  
   (c) \( (2\sqrt{3} + 2i)^5 \)
   (d) \( (1 - i)^8 \)  
   (e) \( \frac{-2}{1+3i} \)

4. Show that \( (a) \ |e^{i\theta}| = 1 \), \( (b) \ e^{i\theta} = e^{-i\theta} \).

5. Plot the set of points \( z = e^{i\theta} \), \( \theta \in [0,2\pi) \), in the complex plane.

6. Find all (real and complex) solutions to \( (a) \ x^3 = 1 \), \( (b) \ x^4 = 1 \), \( (c) \ x^5 = 32 \)

**Mixed Review:**

R41.1. Review Chapter 11 (p 804), # 56

R41.2. Evaluate the following definite and indefinite integrals. Some of the definite ones may be improper, careful.

(a) \( \int \frac{\cos x}{1 + \sin^2 x} \, dx \)  
(b) \( \int \frac{e^{\sqrt{x}}}{\sqrt{x}} \, dx \)  
(c) \( \int \frac{1}{2 - 3s} \, ds \)

(d) \( \int \tan^{-1} x \, dx \)  
(e) \( \int \frac{2^{5x}}{5^{2x}} \, dx \)  
(f) \( \int_0^1 ve^v \, dv \)

R41.3. Use the method of separation of variables to solve the differential equations of the form \( dy/dx = f(y)g(x) \). §9.3: 14, 23

R41.4. (a) Sketch the direction field for the differential equation \( dy/dt = y^2 \).
   (b) Solve the initial value problem \( y' = y^2 \), \( y(1) = 2 \) and highlight this solution in your sketch in part (a).
   (c) Solve the initial value problem \( y' = y^2 \), \( y(1) = 0 \) and highlight this solution in your sketch in part (a).

R41.5. Suppose \( \sum_{n=4}^{\infty} a_n = 3 \). Write a formula for the partial sum \( s_n \) of the series. What is \( \lim_{n\to\infty} a_n \)? What is \( \lim_{n\to\infty} s_n \)?
Level 1: none

Level 2: due Monday Dec 4
Handout “Complex Variables” posted on web: # Part II: 1-6