1.1 (a) \( f'(r) \) measures the rate of change of the total cost as a function of the interest rate \( r \). That is, how much the cost increases per increase in interest rate. Units of \( f'(r) = \$ \). (The interest rate \( r \) is a percentage and is therefore dimensionless - it has no units.)

(b) \( f'(10) = 1200 \) means that if the interest rate is 10% and increases, the total cost increases approximately at a rate of 1200\$ per percentage point. (This approximation becomes worse the bigger the increase in the rate is.)

(c) \( f'(r) \) is always positive: the total cost increases as the rate increases.

1.2 The derivative \( m'(x) \) is the instantaneous rate of change of mass with respect to length, and approximates the mass \( \Delta m \) in a small piece of rod of length \( \Delta x \), per unit length. (This quantity is also referred to as the linear density.) Its units are kg/cm.

1.3 \( s(t) = t^3 - 9t^2 + 15t + 10 \)

(a) \( v(t) = s'(t) = 3t^2 - 18t + 15 = 3(t - 5)(t - 1) \)

(b) \( v(t) = 0 \) when \( t = 1, 5 \)

(c) \( v < 0 \) when \( 1 < t < 5 \)

(d) \( v > 0 \) when \( t > 5 \) or \( t < 1 \)

(e) \( s(0) = 10, s(1) = 17, s(5) = -15, s(8) = 66 \). Total distance travelled = \( (17 - 10) + (17 + 15) + (15 + 66) = 120 \) (see diagram)

1.4 0 (why?)

1.5 see book

1.6 (a) \( \frac{df}{dL} = -\frac{1}{2L^2}\sqrt{\frac{T}{\rho}}, \quad \frac{df}{dT} = \frac{1}{4L\sqrt{T\rho}}, \quad \frac{df}{d\rho} = -\frac{1}{4L\sqrt{T\rho^3}} \),

(b) (i) clearly explain why pitch increases

(ii) Since \( df/dT > 0 \), frequency increases when tension increases.

(iii) Since \( df/d\rho < 0 \), frequency decreases when density increases (heavier string).

1.7 (a) \( C'(x) \) approximates the change in the cost per change in the number of yards produced. By setting the change in the number of yards equal 1, you can interpret \( C'(x) \) as approximating the additional cost needed to producing one more yard. (In economics, the derivative of the cost function is called the marginal cost.)

(b) \( C'(200) = 32 \$ \)/yard. It predicts that the cost of producing the 201st yard is approximately 32 \$.

(c) The actual cost of producing the 201st yard is \( C(201) - C(200) = 32.30 \$ \).
2.1 First we determine $c$ using the information given at one instant: 

$$c = \frac{T}{PV} = \frac{18 \, ^{o}C}{30 \cdot 152 \, KPa \cdot mL} = \frac{3 \, ^{o}C}{760 \, KPa \cdot mL}.$$ 

Now we apply implicit differentiation to relate the rates of change:

$$T' = c(P'V + PV').$$

We know: $V, P, T', V'$ and want to solve for $P'$:

$$P' = \frac{T'/c - PV'}{V} = \frac{1 \, ^{o}C}{hr \cdot 3 \, ^{o}C} + \frac{30 \, KPa \cdot 2mL}{hr} \cdot \frac{760}{3} + 60 \, KPa \cdot \frac{3 \, mL}{152 \, hr} \approx 2.0614 \, \frac{KPa}{hr}.$$

2.2 Rectangle with length $l$ and width $w$ has area

$$A = lw$$

Given: $dl/dt = 5 \, \text{in/min}$, $dw/dt = -7 \, \text{in/min}$. Want $dA/dt$ when $l = 6\, \text{in}$ and $w = 8\, \text{in}$. Apply implicit differentiation to $(*)$ to relate rates of change:

$$\frac{dA}{dt} = l\frac{dw}{dt} + \frac{dl}{dt}w$$

Substitute in given values, to obtain that at that moment

$$\frac{dA}{dt} = -42 + 40 \, \text{in}^2/\text{min} = -2 \, \text{in}^2/\text{min}$$

so the area at that moment is decreasing.

2.3 -0.6 m/s

2.4 1/575 ft/min $\approx 0.00174$ ft/min

2.5 see book

3. For §2.9: 5,7,31,35,41, see answers in book.

38. $\Delta F \approx 4kR^3 \Delta R$ so

$$\frac{\Delta F}{F} \approx \frac{4kR^3\Delta R}{kR^4} = 4 \frac{\Delta R}{R}$$

A 5% increase in radius ($\Delta R = .05R$) will give approximately a 20% increase in blood flow ($\Delta F \approx .2F$).

40a What is the formula for the linear approximation $L(x)$ about $x = a$? Write it down, find it for $f(x) = sin(x)$ about $a = 0$.

4.1 2: 4:

4.2 abs max: 2, abs min -2

4.3,4.4,4.6 see book
4.5 First graph $v(r)$ using fact that it has a double root at 0, a single root at $r_0$. Then use calculus to find maximum.

5.1. (a) Domain: $x \in (-\infty, \infty)$. Graph should show: y-intercept (0, 2). $f \to \infty$ as $x \to \pm\infty$. Zero slope at $P_1(-3/2, 205/16)$ and $P_2(0, 2)$. Increasing for $x \in (-3/2, 0) \cup (0, \infty)$, decreasing for $x \in (-\infty, 0)$. Concave up for $x \in (-\infty, -1) \cup (0, \infty)$. Concave down for $x \in (-1, 0)$. Absolute min: 5/16, at $x = -3/2$. No absolute max. Inflection point $I_1(-1, 1)$, $I_2(0, 2)$.

(c) Domain: $x \in (-\infty, 6]$. Graph should show: Intercepts (0, 0), (6, 0). $f \to -\infty$ as $x \to -\infty$. Infinite slope at $P_1(6, 0)$. Zero slope at $P_2(4, 4\sqrt{2})$. Increasing for $x \in (-\infty, 4)$, decreasing for $x \in (4, 6)$. Concave down everywhere in domain. Absolute max: $4\sqrt{2}$, at $x = 4$. No absolute min. No inflection point.

(d) Domain: all real numbers. Graph should show: odd function and y-intercept (0, 0). Horizontal asymptote: $y = 0$. $f > 0$ if $x > 0$. Increasing for $x \in (-b, b)$, decreasing for $x \in (-\infty, -b) \cup (b, \infty)$. Concave up for $x \in (-\sqrt{3}b, 0) \cup (\sqrt{3}b, \infty)$, concave down for $x \in (-\infty, -\sqrt{3}b) \cup (0, \sqrt{3}b)$. Absolute max: $a/(2b)$ at $x = b$. Absolute min: $-a/(2b)$ at $x = -b$. Coordinates of inflection points: $(-\sqrt{3}b, -\sqrt{3}a/(4b))$, (0, 0), $(\sqrt{3}b, \sqrt{3}a/(4b))$.

(e,f) Graph by superposition, then use calculus to find details.

5.2 Use the fact that $V$ has a double root at $t = 40$.

6.1 §3.7, 14: dimensions 40cm by 40 cm by 20 cm

§3.7, 20: Hint: minimize the distance squared. You must show that your answer minimizes the distance. The closest point is $(\frac{5}{2}, \sqrt{\frac{5}{2}})$

§3.7, 37: see book

6.2. Note: profit function $f(p) = \text{revenue} - \text{cost} = p \ast x - (15000 + 2x)$, where $x$ is given function of $p$, is a quadratic function in $p$, so the profit function is maximal at the point of zero slope. Find it.

7.1-7.2 see book

7.3 §3.9, 60:

§3.9, 67: see book

7.4 §6.2, 97: see book

§6.2, 98: $P(t) = \frac{450.268}{1.12567} (e^{1.12567t} - 1) + 400. P(3) \approx 11, 713.$

7.5 §6.2, 55: see book