Worksheet: Graphing polynomials and other basic functions

These problems should be done without using a graphing calculator, you should not have to spend the time on a calculator to plot these graphs (unless specified otherwise). The rough position of intercepts, asymptotes, behaviour at infinity and points of zero slope should be clear.

1. In this problem you practice graphing translations/dilations of basic functions. This is one of the fundamental graphing techniques.
   (a) \( y = (x - 2)^2 + 2 \)
   (b) \( y = \sin(x - \pi/2) \)
   (c) \( y = 2\cos(x + \pi/2) + 2 \)
   (d) \( (x - 2)^2 + (y + 1)^2 = 1 \)
   (e) \( y = \frac{1}{(x + c)}, \) for any value of \( c \) (plot different scenarios \( c > 0, c = 0, c < 0 \))

2. In this problem you investigate the behaviour of a polynomial near a multiple root.
   (a) Show that for the function \( f(x) = (x - 2)^4 \) the following holds: \( f(2) = f'(2) = f''(2) = f'''(2) = 0 \)
   but \( f^{(4)}(2) \neq 0 \) (ie, the first 3 derivatives are zero at a root of multiplicity 4, but the 4th one isn’t)
   Note: this is true for a root at \( x = a \) of multiplicity \( n \): \( f(a) = f'(a) = f''(a) = \ldots = f^{(n-1)}(a) = 0 \)
   but \( f^{(n)}(a) \neq 0 \)
   (b) Plot \( y = x - 2 \)
   (c) Plot \( y = (x - 2)^2 \)
   (d) Plot \( y = (x - 2)^3 \)
   (e) Plot \( y = (x - 2)^4 \)

3. (a) Explain why the polynomial \( f(x) = x(x - 2)^4 \) looks like \( y = 16x \) near \( x = 0 \) and like \( 2(x - 2)^4 \) near \( x = 2 \). a multiple root.
   (b) On the same graph plot \( y = 16x, y = 2(x - 2)^4 \) and \( y = x(x - 2)^4 \). Use different colors for each graph. You may use the graphing calculator to get an accurate plot.

4. Here you practice plotting polynomials with several roots. The only information you need to use is the position of the roots and the multiplicity of the root, based on what you learnt in problems 2 and 3.
   (a) \( y = (x + 4)(x - 3)^2 \)
   (b) \( y = 10x(x - 1)^4 \)
   (c) \( y = (x - 2)^3 x^2 \)
   (d) \( y = (1 - x)(x - 3)^3(x - 2)^5 \)
   (e) \( y = x^3 + cx, c < 0, c = 0, c > 0 \) (Hint: factor polynomial first, to see what the roots are)

5. The following problem regards graphing functions of \( y \) in the \( x - y \) plane. (We will need this later on in this course.)
   (a) \( x = y(y - 1) \)
   (b) \( x = y^2 - 1 \)
   (c) \( x = y^3 + y \)
   (d) \( x = (y - 3)^2 y(y + 1)^3 \)

6. In this problem you practice graphing superpositions of basic graphs. If a function is a sum of basic functions you know how to graph, graphing the superposition is fairly straightforward and gives you a quick idea of what the function looks like.
   (a) \( y = x + 1/x \)
   (b) \( y = x + \sin x \)
   (c) \( y = x^2 + 1/x \)
   (d) \( y = x^2 + 1/(x - 1) \)