Stat 345: Quiz 2 Based on 2.5-2.6 Name:

Formulas you may need:

1. Independence: Two events are independent if any one of the following equivalent statement is true:
   (1) \( P(A|B) = P(A) \)
   (2) \( P(B|A) = P(B) \)
   (3) \( P(A \cap B) = P(A)P(B) \)

1. A batch of 60 semiconductor chips contains 6 that are defective. Two chips are selected at random, without replacement, from the batch. Let \( A \) and \( B \) denote the events that the first and second chips selected is defective respectively.
   (a) Are \( A \) and \( B \) independent events? Give a proof of your statement.
   
   \[
P(B|A) = \frac{5}{59} \\
P(B) = P(B|A)P(A) + P(B|A')P(A') = \frac{5}{59} \times \frac{6}{60} + \frac{6}{59} \times (1 - \frac{6}{60}) = \frac{6}{60} = \frac{1}{10} \\
P(B) \neq P(B|A), \text{ so } A \text{ and } B \text{ are not independent.}
   \]

   (b) If the sampling were done with replacement, would \( A \) and \( B \) independent? Prove your statement.

   Yes.
   \[
P(B|A) = P(B) = \frac{6}{60} = \frac{1}{10}
   \]

2. In the 2004 presidential election, exit polls from the critical state of Ohio provided the following results:

<table>
<thead>
<tr>
<th></th>
<th>Bush</th>
<th>Kerry</th>
</tr>
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<tbody>
<tr>
<td>no college degree (62%)</td>
<td>50%</td>
<td>50%</td>
</tr>
<tr>
<td>college degree (38%)</td>
<td>53%</td>
<td>46%</td>
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If a randomly selected respondent voted for Bush, what is the probability that the person has a college degree?

Let \( C = \{\text{college degree}\} \) and \( B = \{\text{vote for bush}\} \)

\[
P(C|B) = \frac{P(B|C)P(C)}{P(B|C)P(C) + P(B|C')P(C')} = \frac{.53 \times .38}{.53 \times .38 + .50 \times .62} = .39382
\]