1. (a) Find the first 3 non-vanishing terms of the Taylor series expansion of \( \tan z \) around the origin.
(b) Also find its radius of convergence.

2. Let \( P_1, P_2, \ldots, P_n \) be \( n \) arbitrary points of a plane and let \( PP_k \) denote the distance between \( P_k \) and a variable point \( P \). If \( P \) is confined to the closure of a bounded domain \( \Omega \), show that the product \( \prod_{k=1}^{n} PP_k \) attains its maximum on the boundary of \( \Omega \).

3. Evaluate
\[
\int_{|z|=1} \frac{1 - \cos z}{(e^z - 1) \cdot \sin z} \, dz.
\]

4. Show that \( z^5 + 15z + 1 = 0 \) has one root in the disc \( |z| < \frac{3}{2} \), four roots in the annulus \( \frac{3}{2} < |z| < 2 \).

5. Find the fallacy in the following ‘proof’:

Let \( m \) and \( n \) be two arbitrary integers. Then we have
\[
e^{2m\pi i} = e^{2n\pi i}, \quad \therefore \quad (e^{2m\pi i})^i = (e^{2n\pi i})^i.
\]
It follows that
\[
e^{-2m\pi} = e^{-2n\pi}.
\]
Since \(-2m\pi\) and \(-2n\pi\) are all real, we must have
\[
-2m\pi = -2n\pi, \quad \therefore \quad m = n.
\]
6. Find a conformal mapping of the domain

\[
\left\{ z \in \mathbb{C}; \ |z| < 1, \ \Re z > \frac{\sqrt{2}}{2} \right\}
\]

onto the unit disc.

7. Construct a meromorphic function having simple poles with residue 1 at Gaussian integers \( \omega_{mn} = m + in \) \((m, n \in \mathbb{Z})\).

8. True or false:

(a) If \( w = f(z) \) maps \( \Omega_z \) conformally onto \( \Omega_w \), then \( f'(z) \) never vanishes in \( \Omega_z \).

(b) Any two annuli can be mapped conformally onto each other.

(c) Any two non-intersecting circles can be mapped to a pair of concentric circles by a Möbius transformation.

(d) A bounded (real-valued) harmonic function in the plane \( \mathbb{C} \) is a constant.

(e) If \( \{ f_k(z) \}_{k=1}^\infty \) is a sequence of univalent (one-to-one) analytic functions which converges uniformly on every compact subset of a domain \( \Omega \), to a non-constant function \( f(z) \) in a domain \( \Omega \), then \( f(z) \) is a univalent analytic function in \( \Omega \).

(f) There is a non-constant entire function having both \( 2\pi \) and \( i \) as its periods; i.e.

\[
f(z + 2\pi) = f(z) = f(z + i) \quad \text{for all } z \in \mathbb{C}.
\]

9. Prove or disprove two of the six propositions in the previous problem.