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LINEAR SPIN-UP IN A SLICED CYLINDER

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Spin-up and spin-down in a circular tank with a uniformly sloping bottom are studied experimentally and numerically for small values of the relative change in the angular velocity of the tank. Generally, the initial single-cell flow evolves into a number of smaller vortices. The evolution is compared with an analytical model based on an expansion of the flow field in linear Rossby waves (Pedlosky and Greenspan, 1967). Although it is possible to tune the experimental parameters in such a way that agreement with the theory is found, in most cases the experiments show shedding of vortices in the initial stage of the spin-up or spin-down, a phenomenon not described by the analytical model. Nonetheless, in such cases the analytical model still accounts for other observations: the alternating generation of cyclonic and anticyclonic vortices in the eastern part of the tank and their subsequent westward motion.

Keywords: Spin-up; β-plane; sliced cylinder

1. INTRODUCTION

During the last decades, the spin-up of a homogeneous fluid to a final state of solid-body rotation due to a change in the angular velocity of its container has received considerable attention. Most studies on this

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subject concerned the flow in a circular tank (Greenspan and Howard, 1963; Wedemeyer, 1964; Weidman, 1976; Van de Konijnenberg and Van Heijst, 1995). In a circular geometry, the flow remains approximately azimuthal, and slowly gains a higher angular velocity by a secondary flow driven by the Ekman layers at the bottom and, if present, at the lid of the tank. Van Heijst (1989) first studied spin-up in cylindrical geometries with non-circular cross section, such as a semi-circle and a circle with a radial barrier. This study was followed by a number of publications on spin-up in various geometries, most of them rectangular (Van Heijst *et al.*, 1990; Van Heijst *et al.*, 1994; Suh, 1994; Van de Konijnenberg, 1995). In non-circular geometries, the flow usually separates from the lateral boundaries, and evolves into a pattern of a small number of counterrotating vortices. The organization into a quasi-steady streamline pattern depends on experimental parameters such as the shape of the container and the angular velocity. Geometries that can accommodate a small number of equally-sized vortices, such as a slender rectangular tank, favour the formation of a quasi-steady vortex pattern. The occurrence of self-organization in spin-up experiments is related to the two-dimensional nature of rotating flows. Spin-up flows are not only approximately horizontal, but also independent of the vertical coordinate. However, the presence of Ekman layers leads to differences from purely two-dimensional flows, in particular with respect to the decay of the relative flow.

A different class of non-axisymmetric geometries is formed by the introduction of a sloping bottom. Already in 1967, Pedlosky and Greenspan studied spin-up in a sliced cylinder – a circular cylinder with a uniformly sloping bottom and a flat rigid lid. Pedlosky and Greenspan presented an analytical solution for the time-dependent flow for the case in which the increase in angular velocity is very small. This solution consists of an expansion in Rossby wave modes, multiplied by an overall factor representing the exponential decay of the relative flow due to Ekman pumping.

Van Heijst *et al.* (1994) combined a rectangular geometry with a sloping bottom. In this case the flow tends to become very complicated and erratic; usually there is no formation of a quasi-steady streamline pattern. In late stages of the evolution it should be possible to describe the flow in terms of linear Rossby waves, but unless the relative increase in angular velocity is small compared with the relative

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2. GENERAL BACKGROUND

Three dimensionless parameters d
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is small compared with the relative

differences in depth, separation from the sidewall and subsequent vortex formation takes place in the beginning of the experiment.

In the present paper we study the spin-up in a circular tank with a uniformly sloping bottom. This geometry has the attractive property of axisymmetric flow in the limiting case of a vanishing topography, so that the effect of the topography becomes particularly clear. We will show that also in this case, flow separation and vortex shedding from the sidewall is the rule rather than the exception.

The study of the flow in a tank with a sloping bottom has relevance for meso-scale flows in the Earth's atmosphere and oceans, since the non-uniform depth acts in a similar way as a latitudinal variation in the Coriolis parameter f. Indeed, in Pedlosky and Greenspan (1967) and in subsequent papers by Beardsley (1969) and Beardsley and Robbins (1975) the sliced cylinder had the purpose of modelling the ocean circulation. The emphasis of those papers, however, is on flows with stationary forcing by a differential rotation of the rigid lid. In the present paper we study the spin-up process without aiming directly at the modelling of a particular geophysical flow.

The further organisation of this paper is as follows. Some general aspects of the spin-up flow in the geometry under consideration are reviewed in Sect. 2. The linear theory and the experimental and numerical methods are explained in Sects. 3, 4 and 5. The experimental results are presented in Sect. 6 and discussed in Sect. 7, to be followed by the numerical results in Sect. 8. Finally, some general conclusions are drawn in Sect. 9.

2. GENERAL BACKGROUND

Three dimensionless parameters determine the spin-up we consider in this paper. The first one is the Rossby number Ro, which is, in general, a measure of the relative flow with respect to the background rotation. In the present paper we define it as $Ro = \Delta\Omega/\Omega$, with $\Delta\Omega$ the increase in angular velocity and Ω the final angular velocity. We will restrict ourselves to Rossby numbers in the range between -0.1 and 0.1. The second parameter is the Ekman number E, defined as $\nu/(\Omega H^2)$, with ν the kinematic viscosity and H a measure of the depth in the tank. The third parameter is $\kappa = \Delta H/H_{\max}$, the ratio between the depth difference $\Delta H = H_{\max} - H_{\min}$ between the deepest

and the shallowest part, and the maximum depth. It is this parameter which gives rise to differences from earlier studies on spin-up in a circular tank with a flat bottom. A fourth parameter would be the ratio between depth $H_{\max}$ and radius a of the tank, but in the present paper both $H_{\max}$ and a are fixed.

It is well known that an inclined bottom influences a rotating fluid by vortex stretching, in a way that leads to similar dynamics as in the β-plane model of meso-scale geophysical flow (see e.g., Van Heijst, 1994). Summarizing the essential points, the flow of fluid along a depth gradient leads to an increase of vorticity ω given for small Rossby number by $\partial\omega/\partial t = 2\Omega H^{-1}\mathbf{v}\cdot\nabla H$, where $\mathbf{v}$ is the horizontal velocity field. With the depth gradient taken in the negative y-direction, this can be written as $\partial\omega/\partial t = -\beta v_y$, with $\beta = -2\Omega H^{-1}\partial H/\partial y$. In a geophysical context, one finds the same term $-\beta v_y$, but with $\beta = 2\Omega\cos\phi/R$, with ϕ the latitude and R the Earth's radius. In view of this analogy, the shallow part of the tank is referred to as the north, and the deeper part as the south. In this paper we take the x- and y-direction in the direction of the east resp. the north, the origin coinciding with the centre of the tank.

According to both the topographic and geophysical definition, the parameter β is a local quantity. However, it is common practice to use the β-plane model, in which β is uniform throughout the domain under consideration. In a sliced cylinder this is never quite true, since the depth is not uniform. [A uniform value for β could be obtained by a depth of the form $H(y) = c_1\exp(y/c_2)$, but this is not a geometry we used in our experiments.] The question naturally arises what value for H should be chosen if the sliced cylinder is to be represented by a β-plane. In the present paper we adopted the central value $H(0) = (1/2)(H_{\max} + H_{\min})$, representing an average of the depth over the area of the tank. Although arbitrary to a certain extent, this choice provides a reasonable average between the extreme values $H_{\max}$ and $H_{\min}$. Note that the same question appears in the definition of the Ekman number; also in that case we choose the average depth $H(0)$.

In the experiments, the rotation of the flow does not only give rise to the topographic β-effect, but also to the formation of Ekman layers at the bottom and top of the vessel. The Ekman layers lead to a secondary flow, which ultimately leads to the decay of the relative flow. The secondary flow leads to a contraction or dilatation of the

two-dimensional velocity field, so
dimensional divergence $\nabla\cdot\mathbf{v}$. If b
Ekman number are small, $\nabla\cdot\mathbf{v}$ ma
$\lambda = 2\sqrt{\nu\Omega}/H$ is the inverse of the
Howard, 1963). Making this assu
the vorticity equation becomes

$$\frac{\partial\omega}{\partial t} + \mathbf{v}\cdot\nabla\omega + \beta v_y$$

This equation, together with th
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cal simulation.

More generally, the smallness
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$$\frac{\partial\omega}{\partial t} + \mathbf{v}\cdot\nabla\omega + \beta(1+\omega/2\Omega)v$$

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two-dimensional velocity field, so it can be represented by the two-dimensional divergence $\nabla\cdot\mathbf{v}$. If both the Rossby number and the Ekman number are small, $\nabla\cdot\mathbf{v}$ may be approximated by $\lambda\omega$, where $\lambda=2\sqrt{\nu\Omega}/H$ is the inverse of the spin-up time (see Greenspan and Howard, 1963). Making this assumption and including the β-term, the vorticity equation becomes

$$\frac{\partial\omega}{\partial t}+\mathbf{v}\cdot\nabla\omega+\beta v_y=-\lambda\omega+\nu\nabla^2\omega. \tag{1}$$

This equation, together with the approximation that the two-dimensional velocity field is divergence-free, was used in the numerical simulation.

More generally, the smallness of the Rossby number may be exploited with different theoretical assumptions, all of which are sometimes categorized as 'linear'. First, one may assume that in every point, $\nabla\cdot\mathbf{v}=\lambda\omega$, so the two-dimensional divergence resulting from the Ekman suction is proportional to the relative vorticity. Combined with the 'compressible' (i.e., not divergence-free) two-dimensional vorticity equation, this corresponds to the Wedemeyer model in non-linear spin-up in a circular cylinder (Wedemeyer, 1964). With this model, the vorticity equation becomes

$$\frac{\partial\omega}{\partial t}+\mathbf{v}\cdot\nabla\omega+\beta(1+\omega/2\Omega)v_y=-\lambda(1+\omega/2\Omega)\omega+\nu\nabla^2\omega, \tag{2}$$

where also the β-term is given in its 'compressible' form. In the present paper we use this form of the vorticity equation for qualitative discussions only. Second, one may consider the velocity field as solenoidal in addition to the assumption that $\nabla\cdot\mathbf{v}=\lambda\omega$. This approach reminds of the Boussinesq approximation for flow with density fluctuations, and means that the divergent part of the velocity field is neglected, except in the Ekman damping term where the divergence is replaced by $\lambda\omega$. It can be shown that in order to obtain a consistent model (i.e., one in which the vorticity integrated over the entire domain remains zero), the two factors $1+\omega/2\Omega$ in (2) should be replaced by 1. Thus, this assumption leads to (1); the incompressibility of the two-dimensional flow is explicitly visible from the form of the β-term and the Ekman-damping term, and implicitly taken into

account in the advective term. Third, one may also neglect the advective term. This assumption is quite drastic, since it eliminates the possibility of vortex shedding from domain boundaries; in fact, it eliminates the very existence of vortices in the way they are usually interpreted. In reality, advection of vorticity tends to be important even if the Rossby number is small. Indeed, in the absence of the β-effect and (more hypothetically) Ekman suction, the influence of the background rotation is not noticeable at all, so the advective term plays the same role as in a non-rotating fluid. We will show that in our experiments with the Rossby number of the order of 0.1, the advective term cannot be neglected.

3. LINEAR THEORY

An analytical expression to the linear spin-up problem in the limiting case $Ro \to 0$ was given by Pedlosky and Greenspan (1967). Their result is a solution of (1) with the omission of the advective term, the Ekman damping term and the viscous term. There seems to be little need to review the method they used (for more explanation the reader may also consult Pedlosky, 1987). However, the result of Pedlosky and Greenspan seems to include a typographical error with respect to the time-dependent terms. We repeated the calculation and arrived (in dimensional quantities) at

$$\frac{\psi(x,y,t)}{\Delta\Omega a^2} = -\sum_{m=0}^{\infty}\sum_{n=1}^{\infty}\frac{4\delta_m}{k_{mn}^2}\mathrm{J}_m\left(\frac{k_{mn}r}{a}\right)\cos(m\theta)\cos\left(\frac{k_{mn}x}{a}+\frac{\beta at}{2k_{mn}}-\frac{m\pi}{2}\right) \tag{3}$$

or, with faster convergence,

$$\frac{\psi(x,y,t)}{\Delta\Omega a^2} = \frac{1}{2}\left(\frac{r^2}{a^2}-1\right)+\sum_{m=0}^{\infty}\sum_{n=1}^{\infty}\frac{8\delta_m}{k_{mn}^2}\mathrm{J}_m\left(\frac{k_{mn}r}{a}\right)\cos(m\theta)$$
$$\sin\left(\frac{k_{mn}x}{a}+\frac{\beta at}{4k_{mn}}-\frac{m\pi}{2}\right)\sin\left(\frac{\beta at}{4k_{mn}}\right) \tag{4}$$

with ψ the stream function, and k_{mn} order Bessel function, $\delta_0 = 1/2$, and translation of symbols, this corre Greenspan (1968).

Each mode is characterized by a azimuthal nodal lines, intersected b north–south and moving from eas up to a solid-body rotation. Howev velocity, the coherence of the initi $\beta at = 5.68$, the first invading vortex the tank, replacing the initial anti and smaller structures appear as overview of the first stages of the f to $\beta at = 48$).

4. EXPERIMENTAL SET-UP

The experiments were performed $a = 20$ cm, depth $H_{\max} = 20$ cm and graphic β-plane was created with elliptic plate designed for a partic the tank was placed on a rotating water seeded with small ($\sim 50\,\mu$m) the rigid lid, so there is no free su of the table was changed in a few period of a few seconds is compa the tank, but short compared wit tive flow in the rotating system. Ω was always equal to 2.0 rad/s. T electronically, and are accurate to obtained by intersecting the tank applying tracking techniques to video recording of the flow was m with the tank. Then, after the expe by a PC equipped with a frame g Image system developed by Dal image processing system that allo

erm. Third, one may also neglect the tion is quite drastic, since it eliminates dding from domain boundaries; in fact, e of vortices in the way they are usually tion of vorticity tends to be important is small. Indeed, in the absence of the cally) Ekman suction, the influence of t noticeable at all, so the advective term on-rotating fluid. We will show that in ssby number of the order of 0.1, the ected.

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$$\left.\frac{r}{}\right) \cos(m\theta) \cos\left(\frac{k_{mn}x}{a} + \frac{\beta a t}{2k_{mn}} - \frac{m\pi}{2}\right) \tag{3}$$

$$\sum_{=0}^{\infty} \sum_{n=1}^{\infty} \frac{8\delta_m}{k_{mn}^2} \mathrm{J}_m\left(\frac{k_{mn}r}{a}\right) \cos(m\theta)$$

$$\left.\frac{\beta a t}{k_{mn}} - \frac{m\pi}{2}\right) \sin\left(\frac{\beta a t}{4k_{mn}}\right) \tag{4}$$

with ψ the stream function, and k_{mn} the n-th positive zero of the m-th order Bessel function, $\delta_0 = 1/2$, and $\delta_m = 1$ for $m \geq 1$. With a minor translation of symbols, this corresponds to the formula given by Greenspan (1968).

Each mode is characterized by a stationary pattern of radial and azimuthal nodal lines, intersected by equidistant nodal lines running north–south and moving from east to west. At $t = 0$, the modes add up to a solid-body rotation. However, as each mode has its own phase velocity, the coherence of the initial flow is gradually lost. Around $\beta a t = 5.68$, the first invading vortex cell appears in the eastern part of the tank, replacing the initial anticyclone. More and more vortices and smaller structures appear as time proceeds. Figure 1 gives an overview of the first stages of the flow according to this solution (up to $\beta a t = 48$).

4. EXPERIMENTAL SET-UP

The experiments were performed in a circular tank with radius $a = 20$ cm, depth $H_{\max} = 20$ cm and a transparent rigid lid. A topographic β-plane was created with a false bottom, consisting of an elliptic plate designed for a particular slope. Before the experiment, the tank was placed on a rotating table, and filled with demineralized water seeded with small ($\sim 50\,\mu$m) particles. The tank is filled up to the rigid lid, so there is no free surface. At $t = 0$, the angular velocity of the table was changed in a few seconds from $\Omega - \Delta\Omega$ to Ω. This period of a few seconds is comparable with the rotation period of the tank, but short compared with the rotation period of the relative flow in the rotating system. In the experiments we report here, Ω was always equal to 2.0 rad/s. The angular velocities are controlled electronically, and are accurate to 0.01%. Quantitative results were obtained by intersecting the tank with a horizontal light sheet, and applying tracking techniques to the illuminated particles. First, a video recording of the flow was made with a video camera corotating with the tank. Then, after the experiment, the recording was processed by a PC equipped with a frame grabber. For this purpose, the Dig-Image system developed by Dalziel (1992) was used. This is an image processing system that allows the tracking of particles based

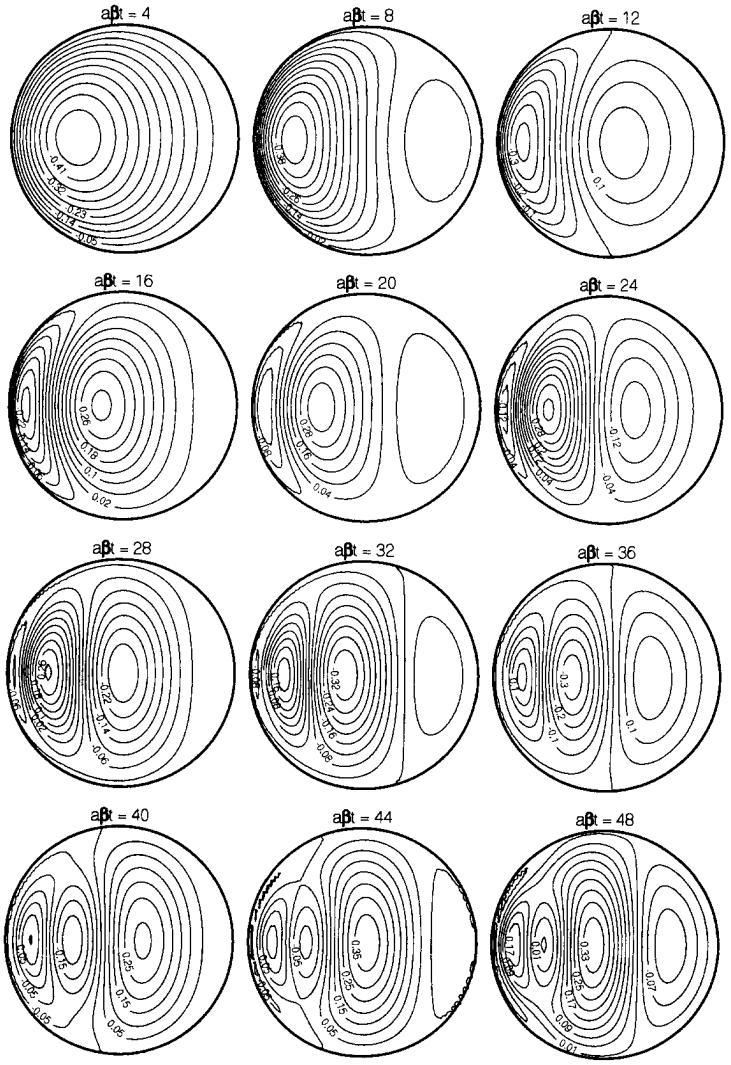

FIGURE 1 Stream function of the spin-up flow according to the analytical solution. The numbers at the contours are the values of $\psi/\Delta\Omega a^2$. In all figures in the present paper, the depth increases in the direction from the top to the bottom of the page. (This means that the positive x-direction is to the right, the positive y-direction to the top of the page.) A positive value of the stream function corresponds to a cyclonic rotation.

on a number of user-defined criteria such as brightness and size. The vorticity was obtained by matching the data with spline functions and manipulating the coefficients of this expansion. The stream

TABLE I Overview of

	$Ro=\Delta\Omega/\Omega$	$\kappa=\Delta H$
Experiment 1	0.01	0.
Experiment 2	0.1	0.
Experiment 3	−0.1	0.
Experiment 4	0.1	0.

function was calculated from the v
In this way the stream function o
velocity field is calculated (not, a
option, the stream function based
the local depth).

In addition to the experiments
performed with a small amount o
water. In this way, a qualitative in
obtained. In particular, these visua
sional instabilities develop at the
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different depths is so convincing
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Four experiments are described
values for the Rossby number (se
a comparison with the analytical
conditions leading to agreement
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These experiments form the back
they demonstrate the phenomena
clarifies the crucial role of the rati

5. NUMERICAL METHOD

In order to make a comparison w
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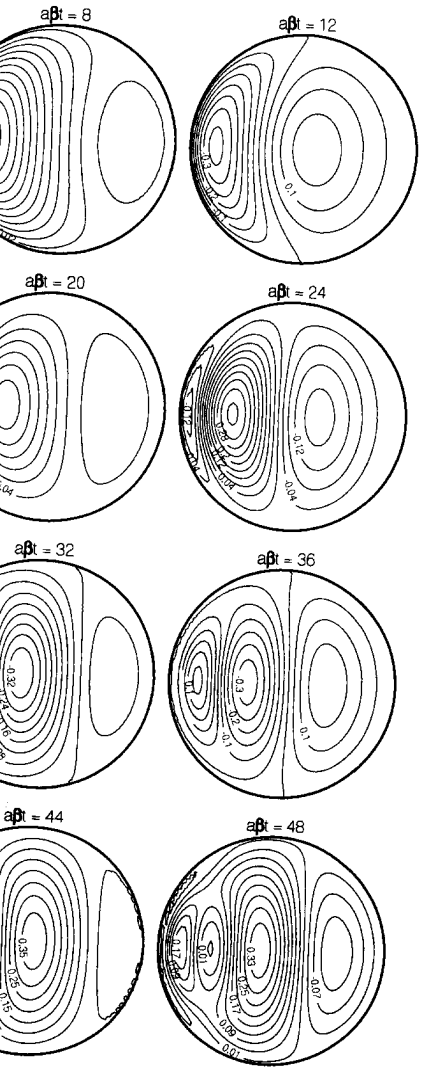

-up flow according to the analytical solution. lues of $\psi/\Delta\Omega a^2$. In all figures in the present from the top to the bottom of the page. (This he right, the positive y-direction to the top of function corresponds to a cyclonic rotation.

iteria such as brightness and size. natching the data with spline func- cients of this expansion. The stream

TABLE I Overview of experimental parameters

	$Ro=\Delta\Omega/\Omega$	$\kappa=\Delta H/H_{max}$	$\beta(m^{-1}s^{-1})$	E
Experiment 1	0.01	0.5	6.66	2.22×10^{-5}
Experiment 2	0.1	0.1	1.05	1.39×10^{-5}
Experiment 3	−0.1	0.1	1.05	1.39×10^{-5}
Experiment 4	0.1	0.5	6.66	2.22×10^{-5}

function was calculated from the vorticity by using a Poisson solver. In this way the stream function of the solenoidal component of the velocity field is calculated (not, as would have been an alternative option, the stream function based on a velocity field weighted with the local depth).

In addition to the experiments with particles, experiments were performed with a small amount of dye added to the otherwise clear water. In this way, a qualitative impression of the flow field could be obtained. In particular, these visualizations show that no three-dimensional instabilities develop at the sidewall, and the flow remains virtually two-dimensional. The absence of differential advection of dye at different depths is so convincing that we considered it to be pointless to perform measurements with light sheets at varying depth.

Four experiments are described in the present paper, all with small values for the Rossby number (see Tab. I). Experiment 1 is aimed at a comparison with the analytical theory. In experiments 2 and 3, the conditions leading to agreement with the analytical theory are relaxed, and vortex shedding from the sidewall is seen to take place. These experiments form the backbone of the paper, in the sense that they demonstrate the phenomena we wish to describe. Experiment 4 clarifies the crucial role of the ratio Ro/κ in experiments of this type.

5. NUMERICAL METHOD

In order to make a comparison with the experimental data, we calculated a numerical solution of (1) with the parameters corresponding to experiment 2 (see Tab. I). For this purpose we used a finite-difference method in the vorticity, stream-function formulation. In this method the flow is assumed to be strictly incompressible, so although the dynamical consequences of vortex stretching and squeezing (induced by the topography and the Ekman layers) are taken into account, the contraction or dilatation itself is neglected. The numerical

scheme utilizes a third-order stiffly stable time-stepping algorithm as described by Karniadakis *et al.* (1991). The diffusive term is treated implicitly, and the circular domain discretized in polar co-ordinates. The use of a staggered grid – i.e., the n grid-points are located at the centres of the n boxes filling the domain, rather than on the edges of the boxes – in the radial direction avoids the need for a special treatment of the co-ordinate singularity at $r=0$. Grid-points were spaced non-equidistantly in the radial direction; a cosine distribution was used across the diameter to have a higher spatial resolution near the wall, where steep gradients appear due to the no-slip boundary condition. The no-slip boundary condition was implemented by adjusting the vorticity at $r=a$ so that the normal derivative of the stream function vanishes. Additionally, in the implicit viscous solver an artificially high value for the viscosity at $r=a$ was used, so that on the outermost grid-points the vorticity was coupled strongly to the vorticity on the wall. In the solution of the Poisson equation, the stream function was taken to be zero at $r=a$. The advective term has the form of a Poisson bracket, and was calculated using the discretized form given by Arakawa (1966). This discretization retains the most important symmetries of the advection term, that is, conservation of energy and enstrophy. The numerical scheme was verified by reproducing the results of the spectral code described in Coutsias and Lynov (1991) for the case of a Lamb dipole impinging on a no-slip wall, while the regularity of the centre was checked by monitoring a Lamb dipole moving through it. No spurious effects were noticed at the co-ordinate origin in that case.

For the initial condition we assume that immediately after a change $\Delta\Omega$ in the angular velocity, the absolute vorticity in the non-rotating system is still unaffected by the motion of the sidewalls. As a result, the relative vorticity of the starting flow has a uniform value $-2\Delta\Omega$. However, at the sidewall, the no-slip boundary condition gives rise to a singular layer of concentrated vorticity with positive sign. This shear layer starts to thicken by viscous diffusion immediately after its formation, but the start of the experiment may be considered as the limiting case in which this shear layer is singular. The velocity field of this initial condition, referred to as the starting flow, is determined by

$$\nabla \times \mathbf{u} = -2\Delta\boldsymbol{\Omega} \tag{5}$$

with zero normal velocity at all sc
cular geometry with a flat bottom
body rotation with angular veloci
bottom, however, an analytic sol
approximation for the starting flo
model was given by Van de Konij
tion αa, this result implies a displa
vortex by $(3/8)\alpha a$ to the deeper pa
tion ΔH of 2 cm, a depth H of 2
yields 0.375 cm. In view of its sm
ignored in the initial condition of
body rotation has been used instea

The representation of the slop
numerical model requires furthe
argument that can be found in P
ing velocity for a layer of flui
$(1/2)(\nu/\Omega)^{1/2}(\cos\alpha)^{-1/2}\omega$ in the no
to a pumping velocity $(1/2)(\nu/\Omega)^{1/}$
ion. This means an increase of th
a factor $\cos^{-3/2}\alpha$. The two-dimen
the sum of the pumping rates at
by the local depth of the fluid. Fo
from 1 by less than 0.2%. In our
taken into account. The depth di
the order of 10% in the local div
of the numerical run with the ir
investigate this matter in further

6. EXPERIMENTAL RESULTS

6.1. Experiment 1: Spin-up $Ro = 0$

Experiment 1 (with the angular
2.00 rad/s and depth varying from
south) was performed in order to
analytical results. The slope is
the Rossby number has about th

stiffly stable time-stepping algorithm as
al. (1991). The diffusive term is treated
omain discretized in polar co-ordinates.
i.e., the n grid-points are located at the
the domain, rather than on the edges
direction avoids the need for a special
singularity at $r=0$. Grid-points were
ne radial direction; a cosine distribution
to have a higher spatial resolution near
ts appear due to the no-slip boundary
ary condition was implemented by ad-
so that the normal derivative of the
ditionally, in the implicit viscous solver
the viscosity at $r=a$ was used, so that
he vorticity was coupled strongly to the
solution of the Poisson equation, the
oe zero at $r=a$. The advective term has
, and was calculated using the discret-
(1966). This discretization retains the
the advection term, that is, conserva-
The numerical scheme was verified by
e spectral code described in Coutsias
se of a Lamb dipole impinging on a
ty of the centre was checked by moni-
through it. No spurious effects were
in in that case.

assume that immediately after a change
absolute vorticity in the non-rotating
e motion of the sidewalls. As a result,
ting flow has a uniform value $-2\Delta\Omega$.
no-slip boundary condition gives rise
ated vorticity with positive sign. This
viscous diffusion immediately after its
experiment may be considered as the
layer is singular. The velocity field of
as the starting flow, is determined by

$$\mathbf{u} = -2\Delta\boldsymbol{\Omega} \qquad (5)$$

with zero normal velocity at all solid boundaries. Clearly, for a circular geometry with a flat bottom, the solution consists of a solid-body rotation with angular velocity $-\Delta\Omega$. In the case of a sloping bottom, however, an analytic solution is difficult to construct. An approximation for the starting flow according to the shallow-water model was given by Van de Konijnenberg (1995). For small inclination αa, this result implies a displacement of the centre of the starting vortex by $(3/8)\alpha a$ to the deeper part of the tank. For a bottom elevation ΔH of 2 cm, a depth H of 20 cm and a radius a of 20 cm, this yields 0.375 cm. In view of its smallness, this displacement has been ignored in the initial condition of the numerical simulation; a solid-body rotation has been used instead.

The representation of the sloping bottom by a two-dimensional numerical model requires further consideration. According to an argument that can be found in Pedlosky (1987), the Ekman pumping velocity for a layer of fluid with uniform inclination α is $(1/2)(\nu/\Omega)^{1/2}(\cos\alpha)^{-1/2}\omega$ in the normal direction, which corresponds to a pumping velocity $(1/2)(\nu/\Omega)^{1/2}(\cos\alpha)^{-3/2}\omega$ in the vertical direction. This means an increase of the bottom Ekman pumping rate by a factor $\cos^{-3/2}\alpha$. The two-dimensional divergence is then given by the sum of the pumping rates at the top and bottom plates, divided by the local depth of the fluid. For $\kappa=0.1$, the factor $\cos^{-3/2}\alpha$ differs from 1 by less than 0.2%. In our simulation, this correction was not taken into account. The depth differences lead to a larger error of the order of 10% in the local divergence, but in view of the success of the numerical run with the incompressible method, we did not investigate this matter in further detail.

6. EXPERIMENTAL RESULTS

6.1. Experiment 1: Spin-up $Ro = 0.01$, $\kappa = 0.5$

Experiment 1 (with the angular velocity increasing from 1.98 to 2.00 rad/s and depth varying from 10 cm in the north to 20 cm in the south) was performed in order to find an optimal agreement with the analytical results. The slope is very pronounced in this case, and the Rossby number has about the smallest value we could deal with.

Since the relative flow is very weak, the accuracy of these results is lower than that of the other experimental results in this paper. However, the stream function, being an integral over the velocity field that is actually measured, still gives acceptable results up to $\beta at = 48$. The results are presented in Figure 2. In spite of the Rossby number being small, its finite value is still noticeable. Around $\beta at = 20$, there is a slight asymmetry with respect to reflection in the x-axis. Experiments with varying $\Delta\Omega$ indicate that this is a result of the Rossby number still being too large. Moreover, at a closer look, the streamline pattern in the experiment seems to evolve slightly faster than the analytical theory predicts. However, this can be ascribed to the unprecise definition of the parameter β in the case of large depth differences (see Sect. 2). As the dimensionless time βat depends on β, the apparent speed of the evolution may easily be off by 10%.

The limited accuracy of this experiment does not allow a detailed quantitative comparison with the theory, but some observations with respect to the value of the stream function can be made. When time increases, the experimental flow becomes weaker than the theoretical solution. This is a consequence of the decay of the experimental flow due to Ekman damping. The spin-up time in this experiment is given in dimensionless time by $\beta a/\lambda = 70.7$. This means that at $\beta at = 48$, the experimental flow should have decayed by a factor $\exp(-48/70.7) = 0.507$. A comparison between the experimental and numerical results (taking into account a time lag $\beta a \Delta t \approx 4$) shows that this is roughly correct. In any case, this issue does not affect the agreement between the experimental and theoretical streamline patterns, since the analytical theory allows for Ekman damping by multiplication the solution with a uniform factor $\exp(-\lambda t)$.

On $\beta at = 4$, however, there is a clear difference between experiment and theory. The experimental flow should have decayed by a factor $\exp(-4/70.7) = 0.945$, but the extremal value 0.35 (we disregard the minus sign) of the stream function in the experiment is smaller than the predicted value $0.49 \times 0.945 = 0.46$. Moreover, the streamlines in the experiment have moved more to the left than in the theory. This difference seems to be connected with the geometrical effect of the sloping bottom. The analytical theory takes the β-effect of the sloping bottom into account, but otherwise the depth is assumed to be

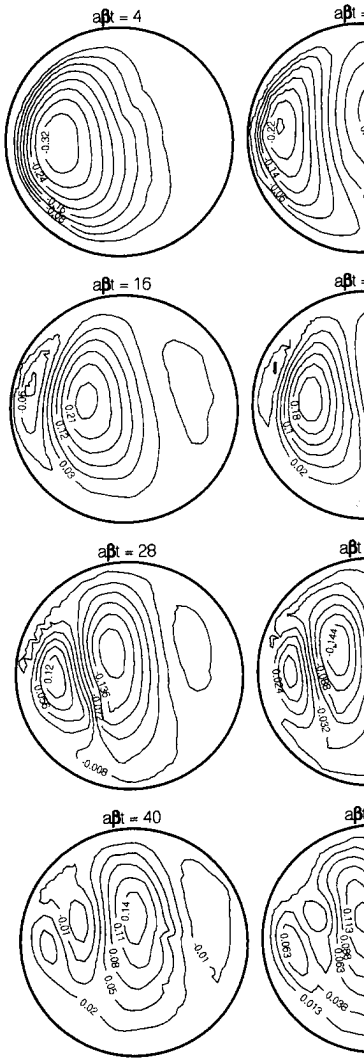

FIGURE 2 Stream function $\psi/\Delta\Omega a^2$ of
from 10 cm to 20 cm, spin-up from 1.98 t

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most pronounced at $t = 0$ (not
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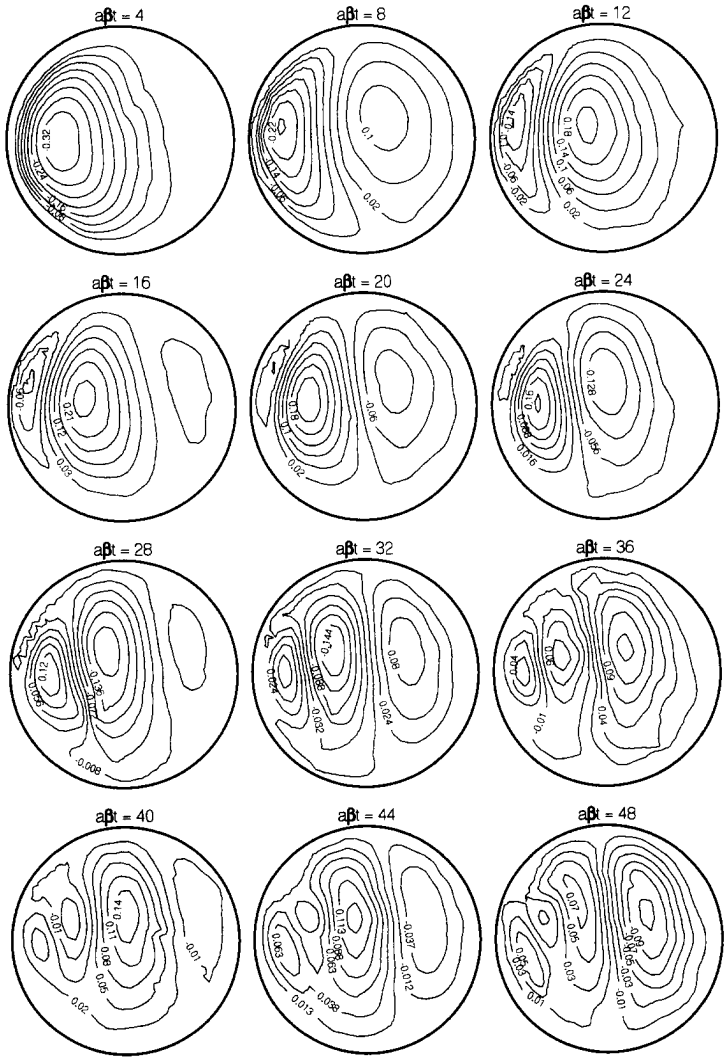

FIGURE 2 Stream function $\psi/\Delta\Omega a^2$ of experiment 1 (radius $a = 20$ cm, depth varying from 10 cm to 20 cm, spin-up from 1.98 to 2.00 rad/s).

uniform; clearly this is not the case in the experiment. This difference is most pronounced at $t = 0$ (not represented in Figs. 1 and 2). The theoretical solution has circular streamlines as initial condition, whereas the experimental flow is asymmetric from the start.

In spite of the mentioned differences, from the viewpoint of a general evolution of the streamline pattern, the qualitative agreement between the experiment and the theory is striking. Thus, the experiment provides a convincing confirmation of the analytical results describing an essential limiting case of the spin-up process.

As the further experiments and the numerical simulations will show, the flow for higher values of the Rossby number is considerably more complex than the analytical results and the experimental data of experiment 1 suggest. This is already forecast by an overview of the time scales appearing in the problem. In the theory, time appears only in the dimensionless combination βat, and we use this product as nondimensionalized time when we present our experimental and numerical results. However, besides the time scale $(\beta a)^{-1}$ one can recognize Ω^{-1} as the formation time of the Ekman layer, $(\Delta\Omega)^{-1}$ as the time needed for an azimuthal displacement of one radian if the starting flow would remain stationary, $H/(2\sqrt{\nu\Omega})$ as the spin-up time scale, and a^2/ν as the time scale associated with viscous diffusion in the horizontal plane. Our scaling facilitates a comparison with the analytical results, but for certain other aspects of the flow (such as the long-term decay of the flow) other choices would be more appropriate.

6.2. Experiment 2: Spin-up $Ro = 0.1$, $\kappa = 0.1$

Next, we consider experiment 2, with $\Omega = 2$ rad/s, $\Delta\Omega = 0.2$ rad/s, and depth varying from 18 to 20 cm. With this experiment we exemplify the dynamics in detail, so we present more comprehensive results for this case than for the other experiments. The experimental data for stream function and vorticity are presented in Figures 3 and 4. For $\beta at < 6$, the flow is close to solid-body rotation. Then, at about $\beta at = 8$, the flow separates from the sidewall. The vorticity distributions clearly show a kind of 'bridge' of cyclonic (dark) vorticity from the sidewall to the vortex, indicating the formation of the vortex from vorticity of the initial shear layer at the sidewall. The principal vortex that is formed by the separation grows in size and strength until it fills almost the entire tank; around $\beta at = 28$, the flow has essentially the opposite sense as at the start of the experiment. Later, this structure is replaced by another vortex formed in the eastern part of the tank.

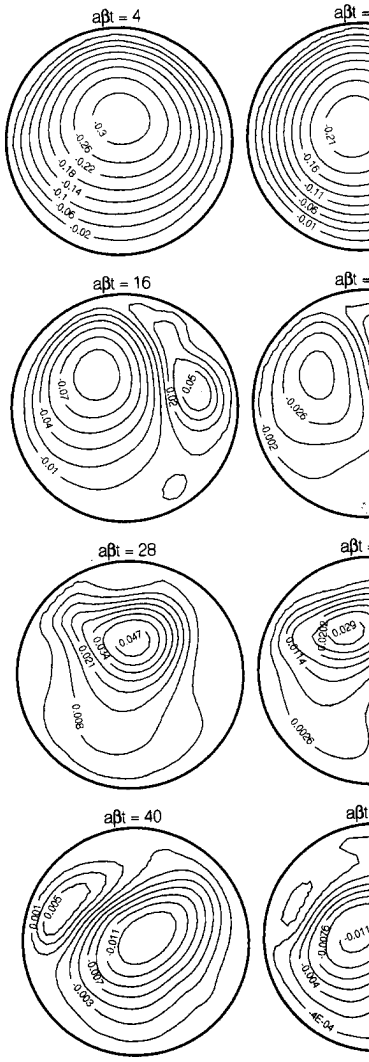

FIGURE 3 Stream function $\psi/\Delta\Omega a^2$ of
from 18 cm to 20 cm, spin-up from 1.8 to

In the period from $\beta at = 24$ to
vortex lose their circular appe
increasingly linear behaviour; wh
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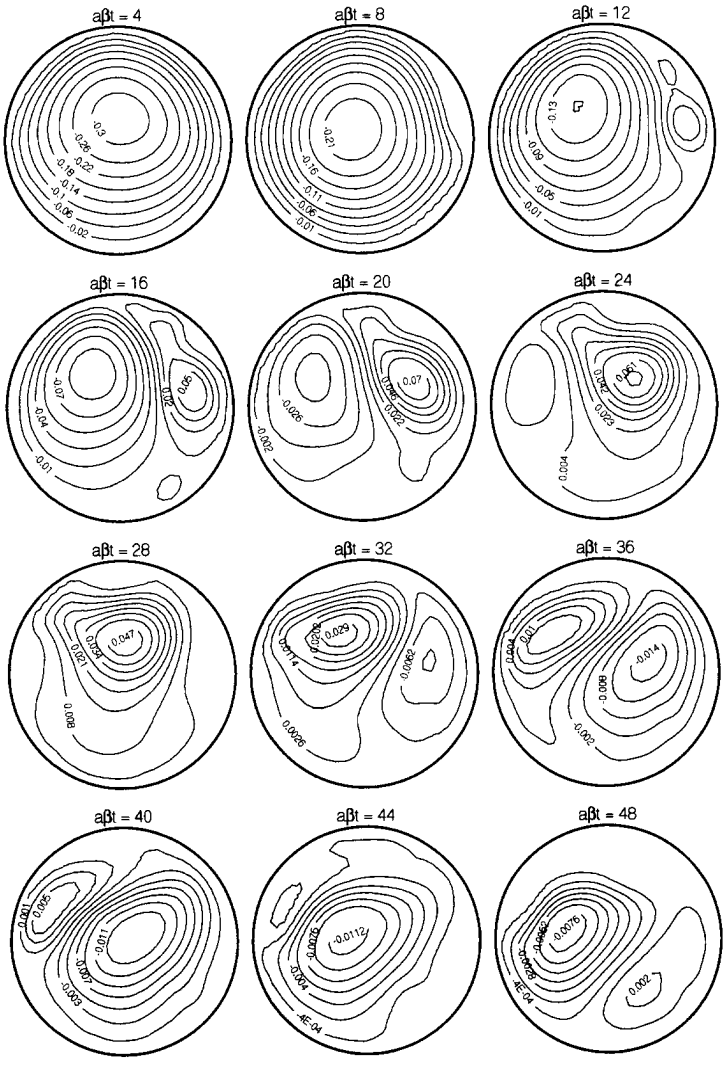


FIGURE 3 Stream function $\psi/\Delta\Omega a^2$ of experiment 2 (radius $a = 20$ cm, depth varying from 18 cm to 20 cm, spin-up from 1.8 to 2.0 rad/s).

In the period from $\beta at = 24$ to 48, the vorticity contours of the shed vortex lose their circular appearance. This is indicative of an increasingly linear behaviour; what one observes is a transition from a strong, nonlinear eddy to a Rossby wave package that is slowly

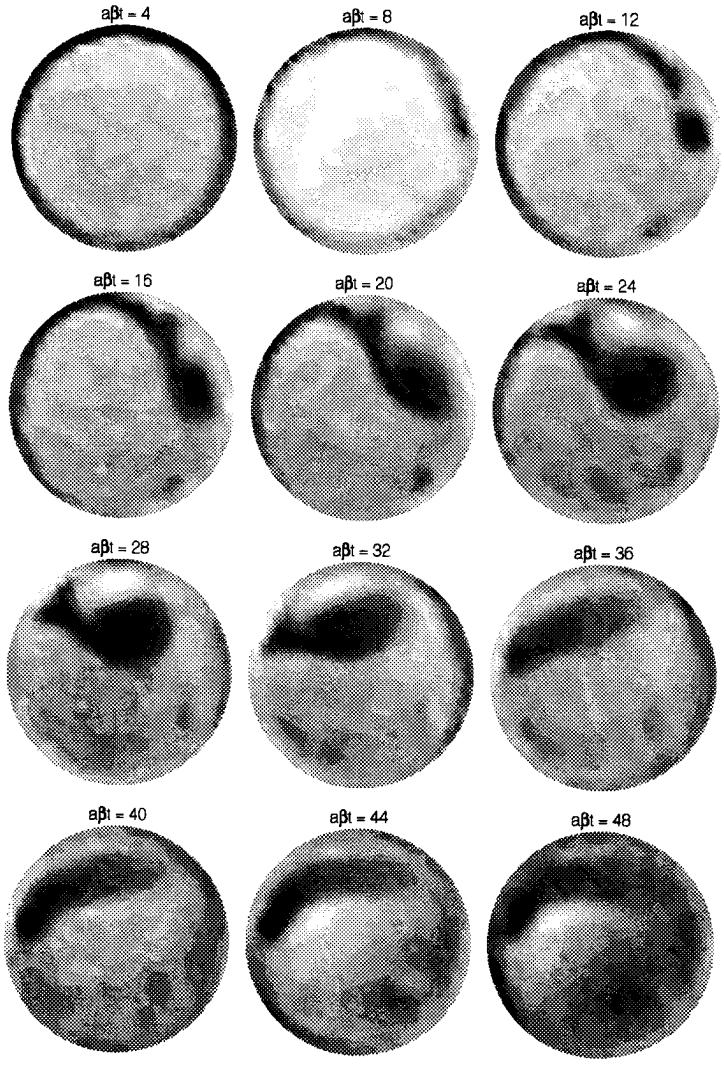


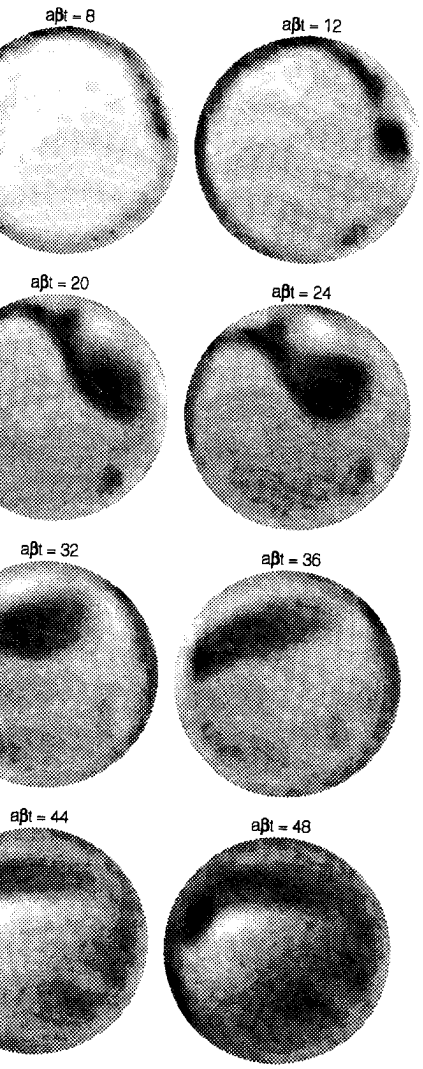
FIGURE 4 Vorticity distribution of experiment 2. The grey scales were chosen to give the best visual impression of the flow, not to facilitate a quantitative comparison between different times. Light corresponds to anticyclonic, dark to cyclonic vorticity. The range of grey scales gradually changes as the relative flow becomes weaker.

dispersed and deformed. The Rossby-wave nature of the flow in later stages can be seen from the general westward motion of the streamline pattern; vortex cells disappear in the western part of the tank, new ones are formed in the eastern part.

6.3. Experiment 3: Spin-down *Ro* =

Experiment 3, spin-down from
bottom elevation of 2 cm, was mea
The basis behind the concept of an
in (1): if the fields $\psi(x,y)$, $u(x,y)$,
ution of (1), then so do $-\psi(x,-y)$
Thus, with the parameters β and
pending on Ω only, one would ex
symmetric results, the symmetry co
to the x-axis.

The results for experiment 3 ar
not perfect, the predicted symmetr
the formation of vortices in the ea
orientation of the separatrix betw
the experiment are encouraging. I
experimental error; in particular,
suggest that the flow in the spin-
faster than in the corresponding
question to what extent symmetr
In general, the symmetry between
for small Rossby numbers. If th
Ekman-damping term in the vor
unavoidably breaking the symmet
In fact, in that case even the sy
down in a circular container wit
cording to the Wedemeyer mode
spin-up and spin-down, the Ekm
$\omega/2\Omega)\omega$. This leads to cyclonic v
clonic vortices, which is indeed ob
0.1 one could estimate the Ek
the experiment by $-0.9\lambda\omega$, for
corresponds to a difference of 2
stage of the experiment. Since th
$a\beta/\lambda$ is equal to 14 for this slope
time lag of the order of 2.8 to
with the experimental results,
4; the streamline patterns for $a\beta$
resemble closely those for $a\beta t$ eq



periment 2. The grey scales were chosen to give
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icyclonic, dark to cyclonic vorticity. The range
elative flow becomes weaker.

ssby-wave nature of the flow in later
eral westward motion of the stream-
ear in the western part of the tank,
rn part.

6.3. Experiment 3: Spin-down $Ro = -0.1$, $\kappa = 0.1$

Experiment 3, spin-down from 2.2 to 2.0 rad/s with a maximal bottom elevation of 2 cm, was meant as the inverse of experiment 2. The basis behind the concept of an 'inverse' experiment is a symmetry in (1): if the fields $\psi(x, y)$, $u(x, y)$, $v(x, y)$ and $\omega(x, y)$ represent a solution of (1), then so do $-\psi(x, -y)$ $u(x, -y)$, $-v(x, -y)$ and $-\omega(x, -y)$. Thus, with the parameters β and ν in (1) held constant and λ depending on Ω only, one would expect experiments 2 and 3 to give symmetric results, the symmetry consisting of a reflection with respect to the x-axis.

The results for experiment 3 are presented in Figure 5. Although not perfect, the predicted symmetry is easy to recognize. In particular, the formation of vortices in the early stage of the experiment and the orientation of the separatrix between large vortices in later stages of the experiment are encouraging. However, the asymmetry is beyond experimental error; in particular, the values of the stream function suggest that the flow in the spin-down experiment decays somewhat faster than in the corresponding spin-up experiment. This raises the question to what extent symmetry-breaking effects can play a role. In general, the symmetry between spin-up and spin-down exists only for small Rossby numbers. If the Rossby number is not small, the Ekman-damping term in the vorticity equation has to be modified, unavoidably breaking the symmetry between spin-up and spin-down. In fact, in that case even the symmetry between spin-up and spin-down in a circular container with a flat bottom would be lost. According to the Wedemeyer model (Wedemeyer, 1964) for nonlinear spin-up and spin-down, the Ekman-suction term is equal to $-\lambda(1 + \omega/2\Omega)\omega$. This leads to cyclonic vortices decaying faster than anticyclonic vortices, which is indeed observed in the experiments. For $Ro = 0.1$ one could estimate the Ekman damping in the beginning of the experiment by $-0.9\lambda\omega$, for $Ro = -0.1$ one finds $-1.1\lambda\omega$. This corresponds to a difference of 20% in the spin-up time in the first stage of the experiment. Since the dimensionless linear spin-up time $a\beta/\lambda$ is equal to 14 for this slope, one would expect a dimensionless time lag of the order of 2.8 to develop. This is in rough agreement with the experimental results, which suggest a time lag of about 4; the streamline patterns for $a\beta t$ equal to 40, 44 and 48 in Figure 3 resemble closely those for $a\beta t$ equal to 36, 40 and 44 in Figure 5.

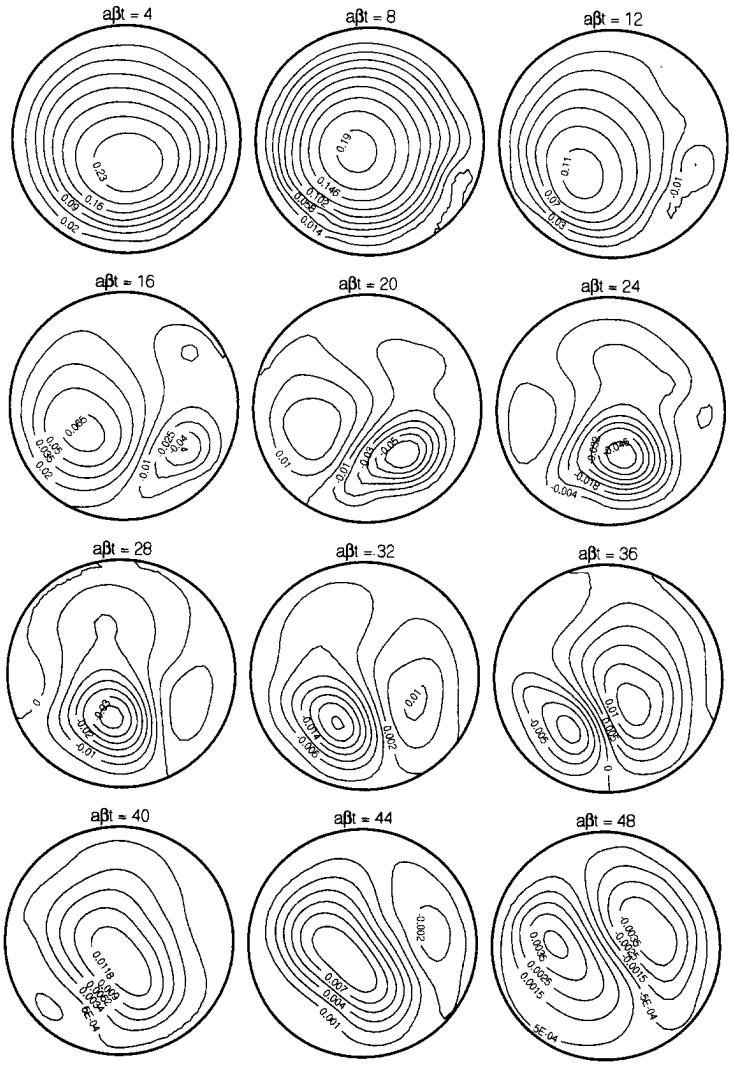


FIGURE 5 Stream function $\psi/|\Delta\Omega|a^2$ of experiment 3 (radius $a = 20$ cm, depth varying from 18 cm to 20 cm, spin-down from 2.2 to 2.0 rad/s).

6.4. Experiment 4: Spin-up $Ro = 0.1$, $\kappa = 0.5$

The results of experiment 4, in which the topography is again extremely pronounced with the depth varying from 10 to 20 cm, are

represented in Figure 6. Separation
a dimensional time measured in sec
fast that we have to allow for an

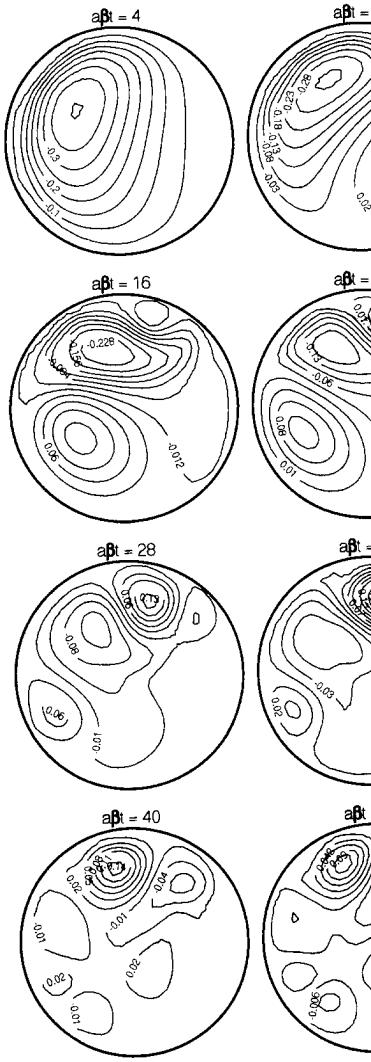
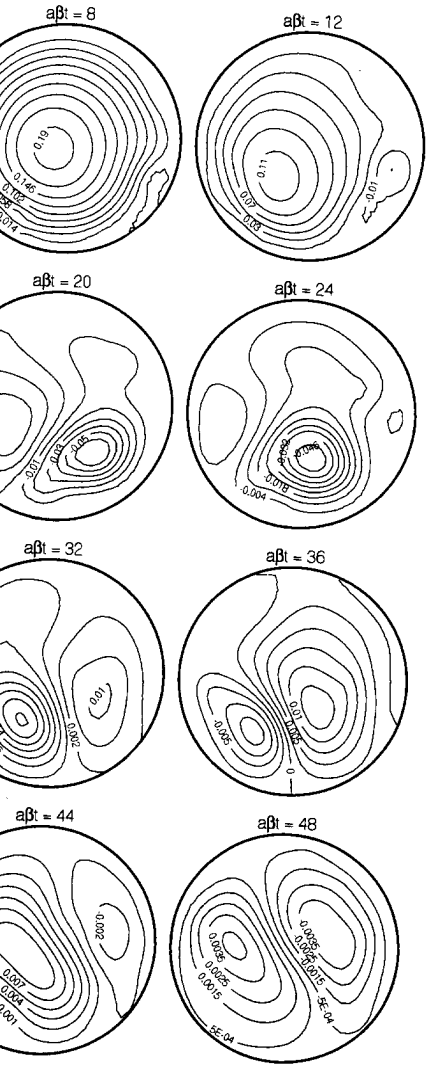


FIGURE 6 Stream function $\psi/\Delta\Omega a^2$ of e
from 10 cm to 20 cm, spin-up from 1.8 to



of experiment 3 (radius $a = 20$ cm, depth varying
.2 to 2.0 rad/s).

= 0.1, κ = 0.5

in which the topography is again
depth varying from 10 to 20 cm, are

represented in Figure 6. Separation from the sidewall takes place in a dimensional time measured in seconds. Indeed, the flow evolves so fast that we have to allow for an uncertainty in the reported time

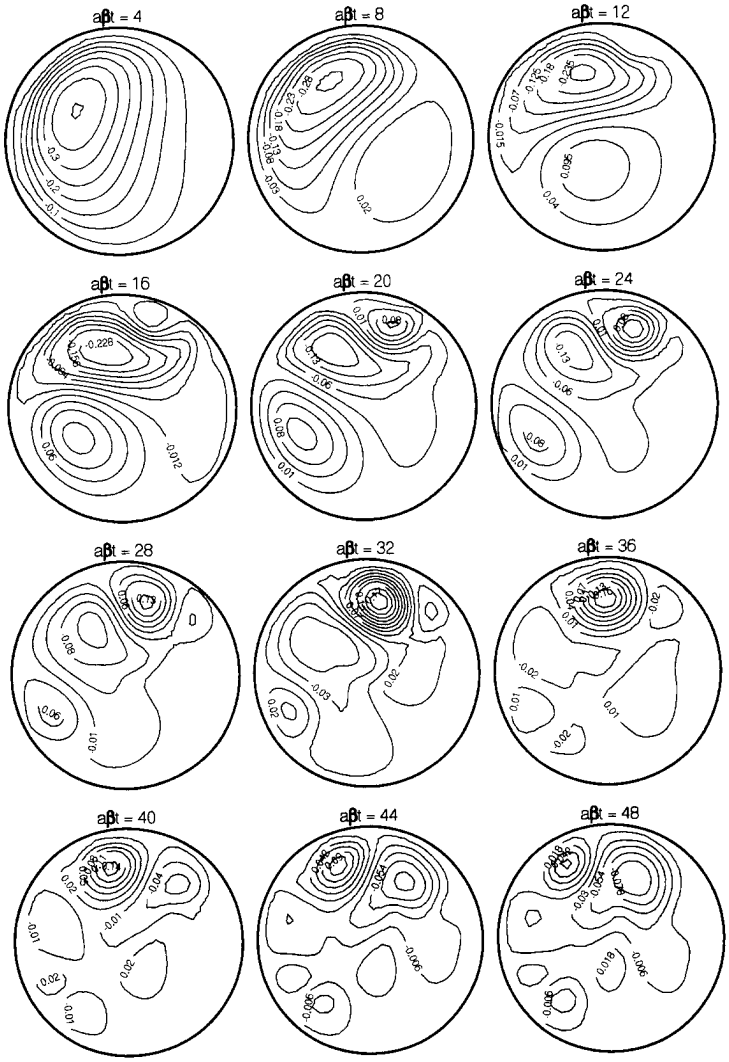


FIGURE 6 Stream function $\psi/\Delta\Omega a^2$ of experiment 4 (radius $a = 20$ cm, depth varying from 10 cm to 20 cm, spin-up from 1.8 to 2.0 rad/s).

of the order of $\beta at = 1$, since it takes several seconds for the table to reach its final angular velocity.

Compared with all other experiments described in this paper, it is striking that experiment 4 shows an anticyclonic rotation of the dipolar structure consisting of the initial anticyclonic cell and the cyclonic cell formed by vortex separation. Again, the only possible explanation for this phenomenon is vortex stretching, caused by the displacement of vortex columns across isobaths; we will return to this matter in the discussion.

In the further evolution of the flow, one can recognize several general properties of the analytical solution that are, at least to a certain extent, also observed in experiments 2 and 3. In the first place, there is a tendency of all vortices to move to the west. This tendency becomes more pronounced as the relative flow becomes weaker. It seems therefore to be plausible that if one would take the streamlines in an advanced stage of the experiment as an initial condition, the analytical model would give a fair description of the flow from that time. Second, the flow is seen to break up into a number of smaller vortices. Unfortunately, the flow decays before really small scales develop. Substituting larger and larger values for βat in the analytical series expression yields a more and more intricate streamline pattern, but as a result of Ekman damping and other experimental limitations we are not able to monitor the flow of experiment 1 for an indefinite time.

7. DISCUSSION OF THE EXPERIMENTAL RESULTS

An important observation in experiments 2, 3 and 4 is vortex shedding from the sidewall in an early stage in the experiment. Vortex shedding consists of the detachment of the initial shear layer from the sidewall, advection of (in the case of spin-up) cyclonic vorticity away from the wall, and the subsequent roll-up into a vortex. This phenomenon is markedly different from the separation and cell formation seen in the analytical results (Fig. 1) and in experiment 1 (Fig. 2). In the analytical results, no sidewall boundary layer with singular vorticity is formed, and no vortex shedding occurs. This is an implication

of the free-slip boundary conditio
the analytical model; vortex shed
advection and viscous diffusion
are present. A further condition
angular displacement in the cours
flow decays on a time scale $\tau =$
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tank is much larger than the Rh
can say that $Ro/\kappa = (\Delta\Omega/\Omega)/(\Delta H$
will disrupt the flow after the slig
ments can hardly move without
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theory, in which $\Delta\Omega/\Omega$ is arbitra
In the opposite limit $a/L_{Rh} \ll 1$,
tank with only slight modificati
most easily seen by considering
small slope; the fluid may then rot
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Fig. 1) and in experiment 1 (Fig. 2).
wall boundary layer with singular vor-
hedding occurs. This is an implication

of the free-slip boundary condition and the missing nonlinearity in the analytical model; vortex shedding can only take place if both advection and viscous diffusion (with no-slip boundary condition) are present. A further condition for vortex shedding is a significant angular displacement in the course of the spin-up process. Since the flow decays on a time scale $\tau = H/(2\sqrt{\nu\Omega})$ this means that $Ro/\sqrt{E}$ should not be small. For $E \ll 1$ this condition is satisfied for all but the tiniest increases in angular velocity. However, one question is whether fluid elements have enough time to travel over appreciable distances, but it is another matter whether they really do so if the finite spin-up time is not a restriction. Suppose that both Ekman damping and viscous diffusion are absent. Then, the nature of the time-dependent flow depends on the ratio between the magnitudes of the advective term and the β-term. If the flow structures are characterized by a typical length scale L and velocity U, then cross-over between the advective term and the β-term takes place for $L_{Rh} = \sqrt{U/\beta}$, which is known as the Rhines length scale. In a spin-up experiment without dissipation, we may put $U = a\Delta\Omega$ and, at the start of the experiment, $L = a$. In the limiting case $a/L_{Rh} \gg 1$, the radius of the tank is much larger than the Rhines length scale; equivalently, one can say that $Ro/\kappa = (\Delta\Omega/\Omega)/(\Delta H/H_{\max}) \ll 1$. In this case the β-effect will disrupt the flow after the slightest spatial displacement; fluid elements can hardly move without inducing alterations in the velocity field of the same order of magnitude as the initial flow. The analytical theory, in which $\Delta\Omega/\Omega$ is arbitrarily small, corresponds to this case. In the opposite limit $a/L_{Rh} \ll 1$, the fluid may travel all around the tank with only slight modifications induced by the β-effect. This is most easily seen by considering a spin-up experiment with a very small slope; the fluid may then rotate over a full 360° without significant deviations from solid-body rotation.

Additional experiments not reported in the present paper show these conjectures to be valid. In particular, in experiments with $Ro =$ 0.01 and κ decreasing from 0.5 to 0, the streamlines tend to become more circular and the agreement with the linear theory is lost. Thus, the value of the Rossby number being small is really an insufficient criterion for the theoretical analysis to apply; the agreement between experiment 1 and the linear theory owes as much to the strong slope as to the smallness of the relative increase in angular velocity.

Now, consider again the spin-up flow according to the analytical theory. Typical of the solution is the decreasing coherence of the wave modes, resulting in the vortex structures becoming smaller with time. This means that the ratio $L/L_{Rh} \ll 1$ will gradually decrease to the point where the advection can no longer be neglected and the linearity is lost. Note, however, that in the presence of Ekman damping this 'turbulent' stage may never be reached. The Rhines length scale is based on a velocity scale $a\Delta\Omega$, which in the case of Ekman damping should be regarded as an exponentially decreasing quantity. It is therefore possible to perform experiments for which the analytical model (apart from a factor representing the decay of the flow) gives a good representation of the flow at all times. Such a case is shown by experiment 1 (Fig. 2).

It is difficult to give a detailed explanation for when and where the vortex shedding in experiments 2, 3 and 4 (Figs. 3–6) takes place. An elementary consideration is that in the case of spin-up, the shed vortex must be cyclonic, since it consists of the vorticity from the initial shear layer at the wall. Second, one might estimate separation to occur at roughly the same time as in the analytical model, differences appearing by advection of vorticity from the boundary into the interior. However, if $\Delta\Omega/\Omega$ and $\Delta H/H$ are of the same order of magnitude, the separation may occur somewhat later than in the analytical model, since a larger azimuthal displacement is needed to accumulate a change in the relative flow. It seems that this effect is indeed seen in the experiments.

A related issue is the retrograde drift of the shed vortex in experiment 4 (Fig. 6). Apparently, in experiment 4 the value of the parameter Ro/κ is such that the flow is affected strongly by the topography, but only after a rotation of the initial flow over an angle of the order of 180°. Considering the flow as the sum of a solid-body rotation plus a disturbance caused by the vortex stretching, one finds a centre of positive vorticity emerges in the south, and a centre of negative vorticity in the north in that case. Of course, the same topographic vortex stretching is present in experiments 2 and 3, but in that case the disturbance of the relative flow is five times as weak, which is insufficient to disrupt the initial flow within one revolution. On the other hand, if Ro/κ is smaller, the disturbance develops so fast that the vortex-stretching argument applies to small angular

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8. NUMERICAL RESULTS

In order to obtain a better under
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displacements only; this is the parameter regime described by the analytical theory.

Another aspect of the flow is the position of the centre (i.e., the point with zero velocity) in the first stage of the experiment. After a fraction of a revolution of the flow in the rotating system, the flow field consists of the initial solid-body rotation, plus a weak dipole with the positive lobe in the east and the negative lobe in the west. Adding these two contributions, one finds a velocity field in which the centre has shifted slightly to the west. This conclusion corresponds to the theory of Pedlosky and Greenspan (1967). However, in experiments 2, 3 and 4 the angular displacement does not remain small. After half a revolution ($\Delta\Omega t = \pi$), the velocity field may be approximated by a solid-body rotation, plus a dipole with the negative lobe in the north and the positive lobe in the south. This implies a shift of the centre to the north, in agreement with the experimental observations. Note that the northward shift is not caused by a rotation of the initial dipole, but is really explained by vortex stretching, as expressed by the β-term in the vorticity equation.

8. NUMERICAL RESULTS

In order to obtain a better understanding of the spin-up process under consideration, we made a numerical simulation with the parameters corresponding to experiment 2 (Figs. 2 and 3). The results for the stream function and vorticity of the numerical run are presented in Figures 7 and 8. Both the streamline contour plot and the vorticity distributions show a good agreement between the experimental and numerical data, another indication of the two-dimensional nature of the flow and the validity of the two-dimensional representation of the Ekman layer. However, it should be noted that the conditions are favourable for such a comparison; the Rossby number is small, the topography is moderate, and with a Reynolds number (defined as $Re = a^2\Delta\Omega/\nu$) of 8000 the flow lies well within the application range of the numerical code.

Unfortunately, the resolution of our experimental method is insufficient to give a detailed insight in the evolution of the flow at early times. The numerical simulations do not have this limitation,

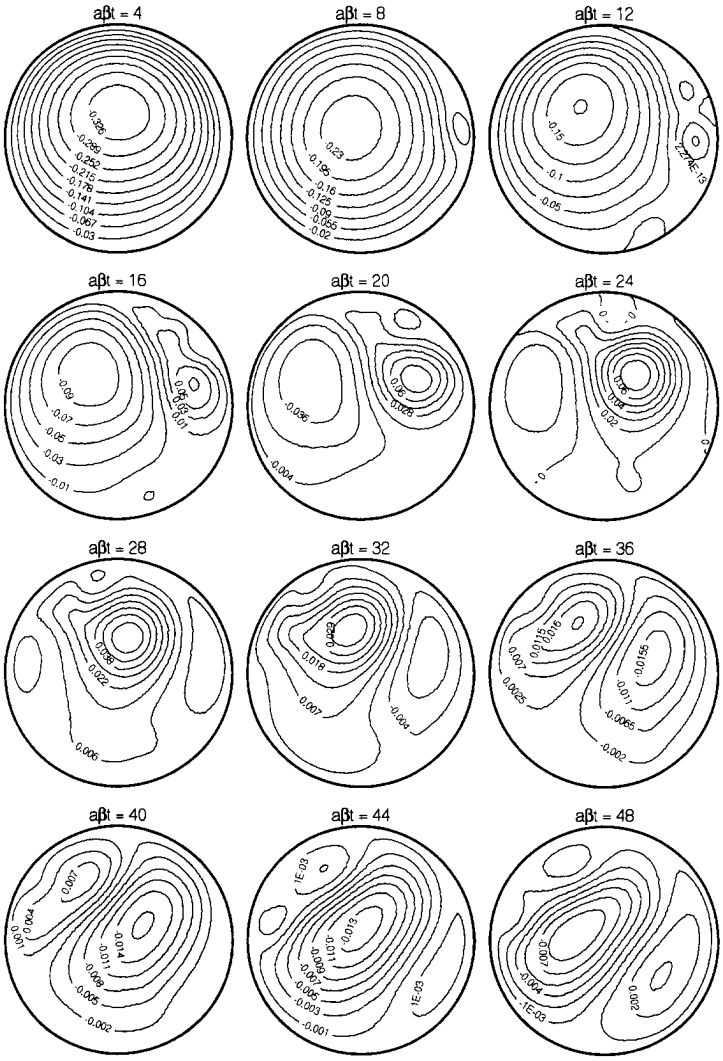

FIGURE 7 Stream function $\psi/\Delta\Omega a^2$ according to the numerical simulation of experiment 2.

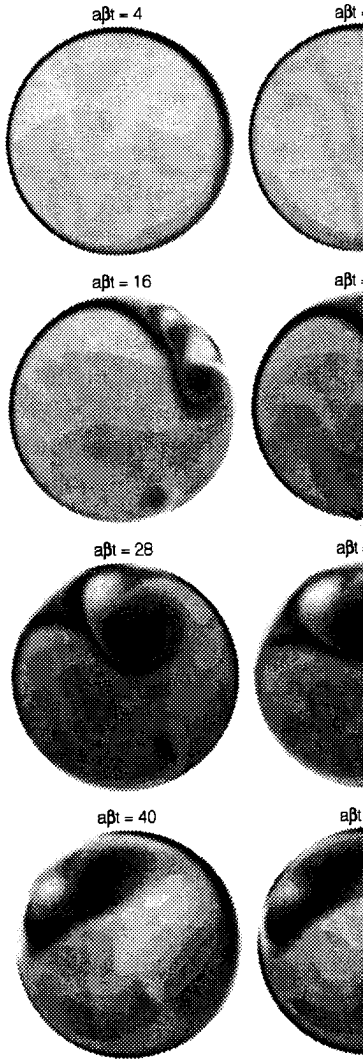

FIGURE 8 Vorticity of the nur

and can be used to study the separation and vortex shedding more closely.

First, we determined the separation time, according to the criterion that $\partial v_\theta/\partial r$ be zero at the wall. In order to distinguish between the

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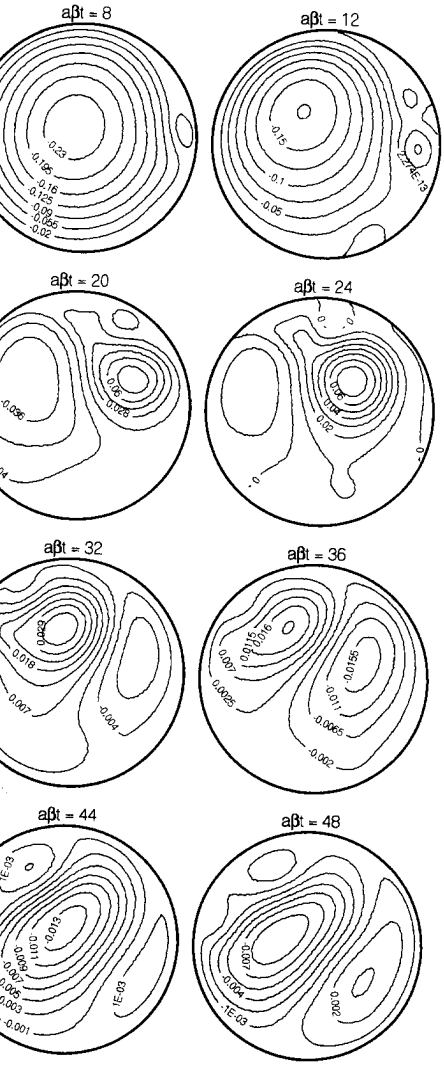

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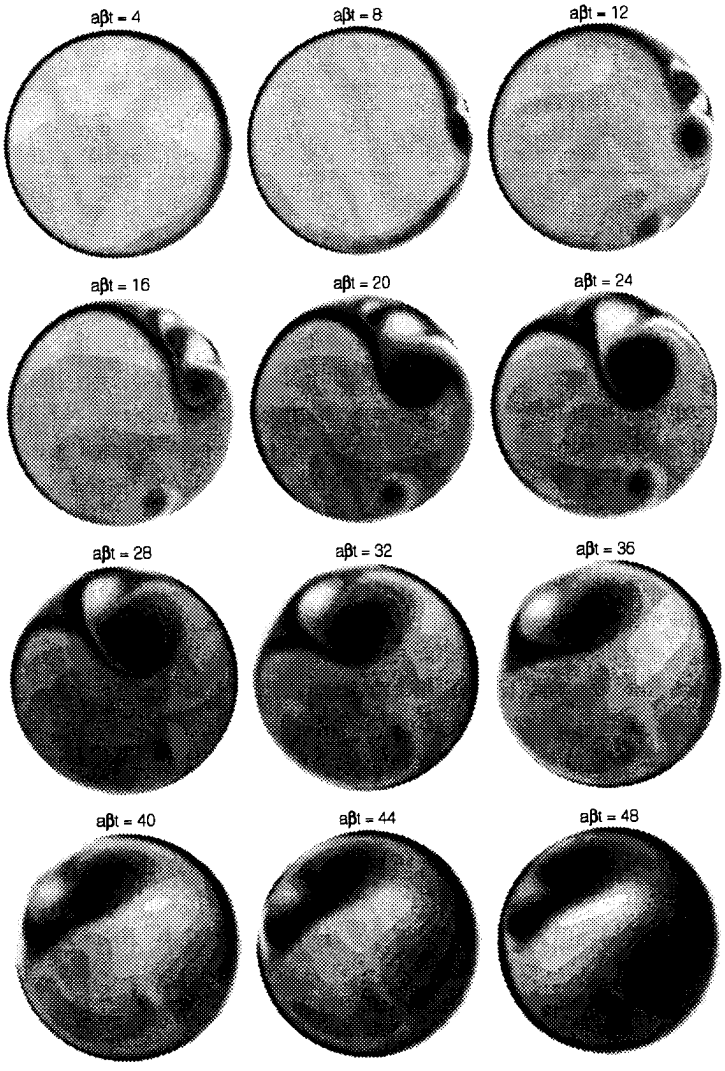

FIGURE 8 Vorticity of the numerical simulation of experiment 2.

influence of different terms in the vorticity equation, we calculated the separation time in three numerical simulations, represented in Table II. All three experiments concern the parameters of experiment 2. Simulation A (the physical run) represents the modelling discussed in

TABLE II Numerical simulations with the parameters of experiment 2

	Designation	*Diffusion*	*Advection*	*Separation time* $\beta a t_{sep}$
Simulation A	Physical run	Yes	Yes	4.19
Simulation B	Inviscid run	No	Yes	8.16
Simulation C	Linear run	No	No	5.69 (analytical model: 5.68)

Sect. 5. In simulation B (the inviscid run), the same parameters are used as in the physical run, but with the diffusive term switched off. In simulation C (the linear run) the advective term is omitted in addition; apart from the uniform damping provided by Ekman pumping, this brings us back to the analytical model. For the physical run we found $\beta a t_{sep} = 4.19$, for the inviscid run $\beta a t_{sep} = 8.16$ and for the linear run $\beta a t_{sep} = 5.69$. The agreement of the linear run with the analytical solution is as expected, and provides a further verification of the correctness of both methods. According to the physical run, the separation time is shorter than in the analytical theory. Apparently, the inward diffusion of positive vorticity accelerates the process of cyclonic vortex formation. The inviscid run indicates that advection tends to delay separation, which is not surprising in view of the fact that the rotating fluid motion will tend to remove concentrations of cyclonic vorticity away from the eastern part of the tank.

Second, we estimated the time at which cyclonic vortices are formed at the sidewall. As a first criterion for the presence of cyclonic vortices, we require the flow to form closed streamlines with $\psi > 0$. In addition, we use a criterion based on the Weiss field (Weiss, 1991), defined as $(1/4)\ (\sigma^2 - \omega^2)$, with ω the vorticity and $\sigma^2 = (\partial v_x/\partial x - \partial v_y/\partial y)^2 + (\partial v_x/\partial y + \partial v_y/\partial x)^2$ a measure of the strain rate. The Weiss field is required to exceed a somewhat arbitrarily chosen, but very small negative threshold. As a consequence of this second criterion, we find only vortices associated with a local concentration of vorticity, not the vortex cells formed by Rossby-wave propagation. Figure 9 shows the vortex-shedding time *versus* the Rossby number for a number of different values of the slope (non-dimensionalized as $a\beta/\Omega$). For $Ro < 0.05$, we find no separation according to the criteria mentioned above, so the flow is in the Rossby-mode regime. For $0.05 < Ro < 0.1$ a vortex-shedding time is found which is roughly inversely proportional to the Rossby number. This dependence would be in contrast with the analytical theory (according to which the flow field depends on $\beta a t$ only) but is in agreement with the

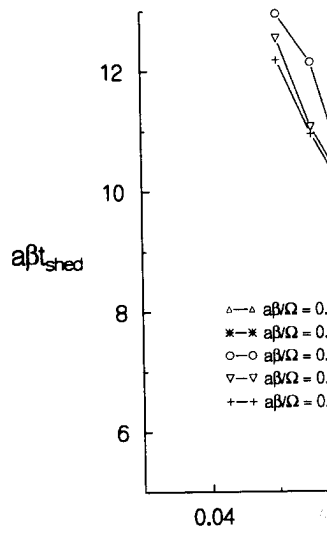

FIGURE 9 Vortex-shedding time accordi
spin-up flows with varying Rossby number

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9. CONCLUSION

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ffusion	*Advection*	*Separation time* $\beta a t_{sep}$
Yes	Yes	4.19
No	Yes	8.16
No	No	5.69 (analytical model: 5.68)

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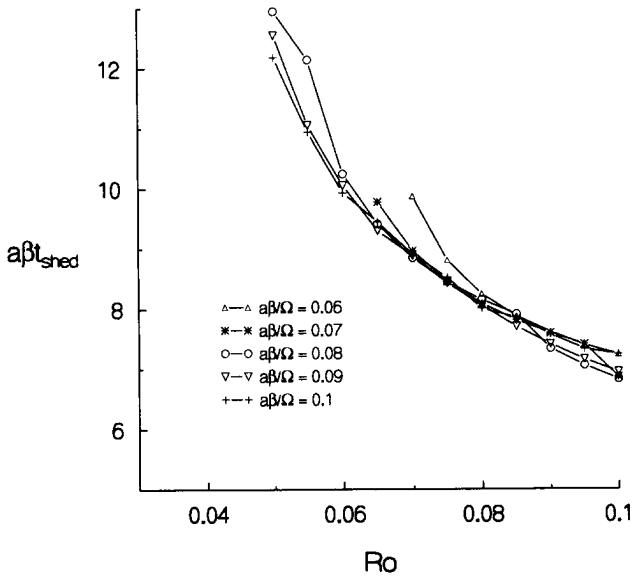

FIGURE 9 Vortex-shedding time according to additional numerical simulations of spin-up flows with varying Rossby number and slope.

notion that for finite Ro/κ and given slope, the fluid has to rotate over a certain angle before the topography becomes noticeable. Moreover, this argument also explains why the scaled vortex-shedding time is, in a reasonable approximation, independent of $a\beta/\Omega$. If the extent to which the flow is affected by the topography is measured by the number of isobaths that is crossed, an increase of the slope by a factor two would make the vortex shedding time twice as short, so the time scale $a\beta t_{shed}$ would be unaltered.

9. CONCLUSION

We have performed four spin-up experiments in a circular tank with a sloping bottom, all with values of the Rossby number in the range from -0.1 to 0.1. The results confirm the validity of theoretical results of Pedlosky and Greenspan (1967), but also indicate that important deviations can occur for conditions that at first sight would seem to meet the criteria on which the theory is based. The theory departs from an equilibrium between $\partial\omega/\partial t$ and βv_y, and consists of

an expansion of the flow in linear Rossby waves. We have shown in Sect. 6.1 that it is indeed possible to tune the parameters of experiments in a rotating tank in such a way that agreement with the linear theory is found. However, the theory does not incorporate viscous diffusion, the formation of a boundary layer at the sidewall, and advective transport of vorticity. The experiments in Sects. 6.2, 6.3 and 6.4 show that even for small values of the Rossby number, vorticity from the initial sidewall boundary layer may enter the domain and roll up into vortices. In earlier work, this phenomenon of vortex shedding has been confused with the inviscid flow separation following from the analytical results. Vortex shedding can be seen in Figure 2.16 of Greenspan (1968), but apparently it was not recognized as such.

Thus, the Rossby number is not the only parameter that plays a role. Equally important for the nature of the spin-up is the ratio Ro/κ (with κ a measure of the depth gradient). If Ro/κ is small, the topography dominates the dynamics, and the analytical theory applies. If Ro/κ is not small, the influence of the topography is moderate, and the validity of the analytical results is lost. In such cases one may expect either ongoing circular motion, or shedding of vortices from the sidewall and the formation of filaments of concentrated vorticity. This argument can also be expressed in terms of the parameter a/L_{Rh}, the ratio of the radius a of the tank and the Rhines length $L_{Rh} = \sqrt{U/\beta}$. Since a/L_{Rh} is inversely proportional to $\sqrt{Ro/\kappa}$, the ratio a/L_{Rh} should be large in order for the theory to be valid.

Our measurements are confirmed by a numerical simulation of one of the experiments in which vortex shedding occurs. This calculation is based on a two-dimensional representation of the flow field, with a linear modelling of Ekman pumping. Owing to the smallness of the Rossby number, this approach is very accurate, and leads to an excellent agreement with the experimental data. By the subsequent omission of the viscous and the advective term in the vorticity equation, it is confirmed that the differences between the linear theory and the experiments are caused by those terms. Further numerical simulations have been used to study the mechanism of vortex shedding in more detail. The time at which vortex shedding occurs is different from the separation time from the linear theory, but is in agreement with predictions based on the notion that Ro/κ has a finite value.

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The decay of the relative flow takes place by viscous diffusion in the horizontal plane and, usually more important, by the Ekman pumping mechanism. In the case of a flat bottom (or top) plate and small Rossby number, the properties of the Ekman layer are well known. The divergence of the two-dimensional flow field can then be coupled to the vorticity, and the flow may be modelled by (1). In the case of experiment 2, this approach leads to a good agreement between experimental and numerical results. Unfortunately, a generally acceptable model for the two-dimensional representation of nonlinear Ekman layers has never been formulated. A consistent model results from the assumption of a linear relationship between vorticity and two-dimensional divergence. In combination with a compressible two-dimensional flow field, this approach is known as the Wedemeyer model. This model describes an inward moving front in the case of nonlinear spin-up, whereas in nonlinear spin-down the flow remains in solid-body rotation. The Wedemeyer model is known to be inaccurate for quantitative applications, but it retains its qualitative validity beyond the conditions it was meant for. In some additional nonlinear spin-down experiments not reported here, it was observed that the initial shear layer remains attached to the sidewall, and the flow remains roughly axisymmetric regardless of the sloping bottom.

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ON THE NORMAL M
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Yu. N. S

Centro de Ciencias de la Atmósfer
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The normal mode instability of harmonic v
rotating sphere is analytically studied. By
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Keywords: Normal mode instability; Leg
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1. INTRODUCTION

In this paper we consider the no
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