

505 Set 3, Part A

October 27, 2012

Write a program to evaluate the following integrals using the trapezoidal rule with uniform spacing:

1. $A = \int_0^1 \sqrt{x} dx = 2/3$

2. $B = \int_0^1 \frac{dx}{1+x^2} = \pi/4$

3. $C = \frac{1}{\pi} \int_0^\pi \cos [(8 \sin x) - x] dx = J_1(8) = 0.234636347$

4. $D = \int_0^1 \frac{dx}{1 + \frac{1}{2} \sin 2\pi x} = 1.15470054$

5. $E = \int_0^1 e^{x^2(1-x)^2} = 1.03414105$

Compute tables of approximate values A_N, \dots, E_N using N subintervals $N = 1, 2, 4, 8, 16, 32, \dots$ and apply Richardson extrapolation. Why is the simple trapezoidal rule so accurate for C and E (integrands not periodic)? Use high enough precision to get the results to the accuracy specified. Explain carefully your findings for all cases. Which case conforms most closely with your expectations? Why?