316-TEST 1 prep

Name:_____

October 5, 2008

Problem	grade
1	
2	
3	
4	
5	
Total	

(1) (20 pts) Solve the initial value problem and sketch the phase plane. Draw in the solution curve corresponding to the given initial conditions.

$$\frac{d}{dt}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3 & -2 \\ 2 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}, \begin{pmatrix} x(0) \\ y(0) \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}.$$

(2) (20 pts) Solve the initial value problem and sketch the phase plane. Draw in the solution curve corresponding to the given initial conditions.

$$\frac{d}{dt}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 6 & 3 \\ 7 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}, \begin{pmatrix} x(0) \\ y(0) \end{pmatrix} = \begin{pmatrix} 2 \\ -2 \end{pmatrix}.$$

(3) (20 pts) Give the type and stability of the point at the origin for the following systems:

1.

$$\frac{d}{dt}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 & -1 \\ 3 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} .$$

2.

$$\frac{d}{dt}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 & -4 \\ 4 & -7 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} .$$

3.

$$\frac{d}{dt}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3 & -2 \\ 4 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} .$$

4.

$$\frac{d}{dt}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 & -5 \\ 1 & -2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} .$$

(4) (20 pts) Solve the initial value problem

$$y' - y = 1 + 3\cos t \ ; \ y(0) = y_0 \ .$$

Find the value of y_0 for which the solution remains finite as $t \to \infty$.

(5) (20 pts) Solve the initial value problem

$$y' = xy^2 \left(1 + x^2\right)^{-1/2} \ ; \ y(0) = 1 \ .$$

Give the maximum interval around $x = 0$ for which the solution to this IVP exists.