The Electric and Magnetic fields

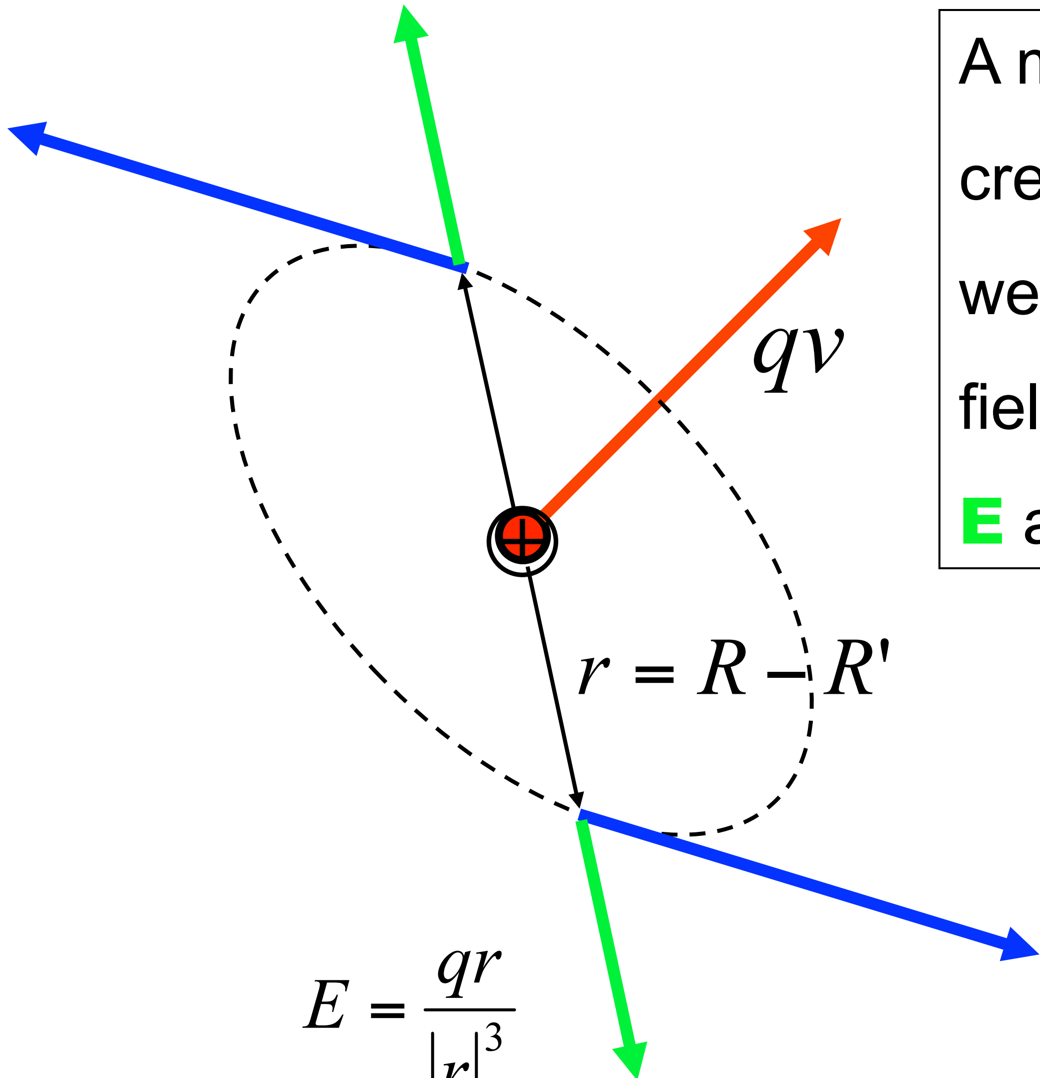
Maxwell's equations in free space

References:

Feynman, Lectures on Physics II
Davis & Snyder, Vector Analysis

Sources: elementary charges

A moving charge **q** creates an electric as well as a magnetic field around itself, **E** and **B**

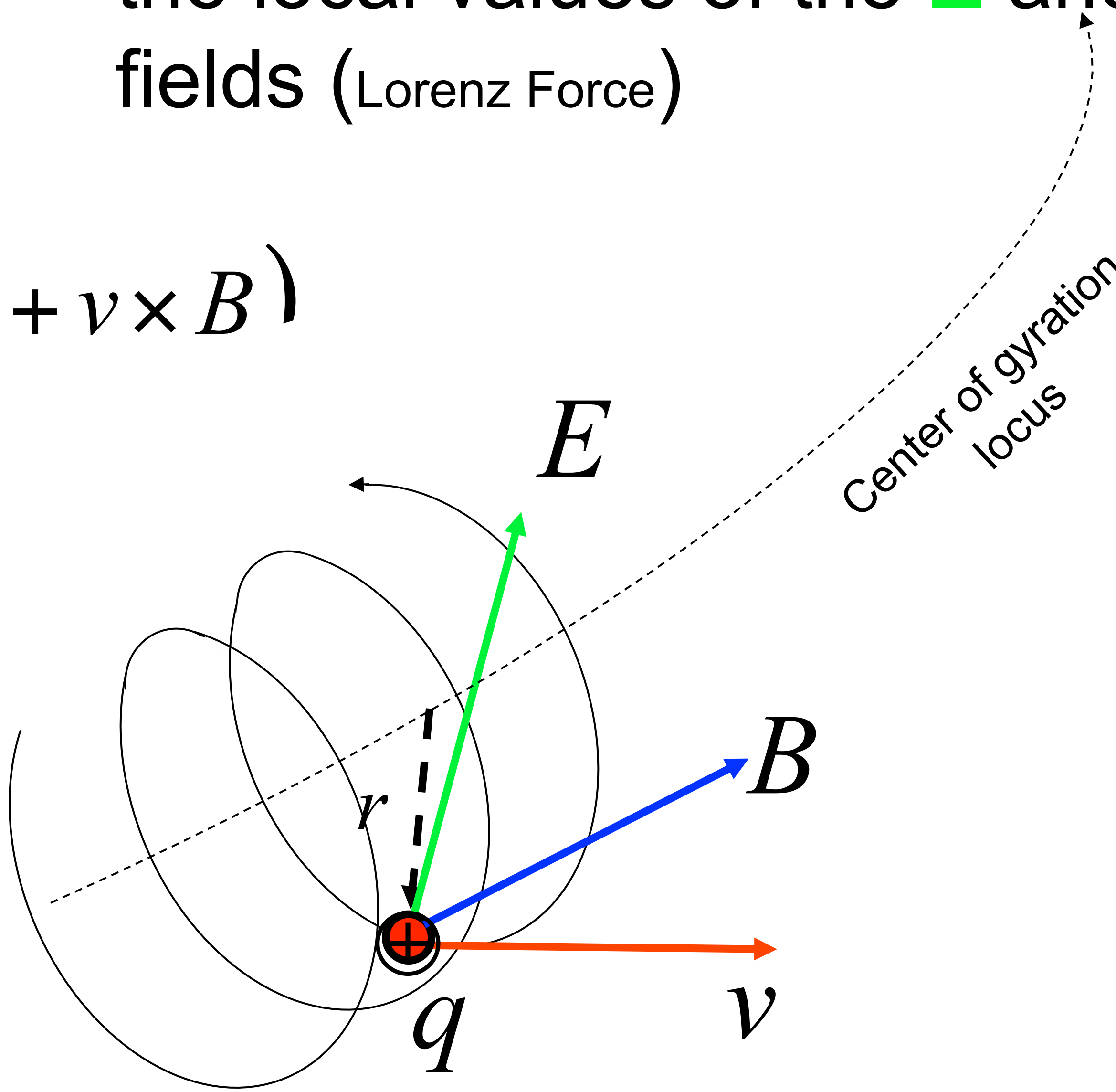


$$B = \frac{qv \times r}{|r|^3}$$

$$E = \frac{qr}{|r|^3}$$

A moving charge **q** is affected by the local values of the **E** and **B** fields (Lorenz Force)

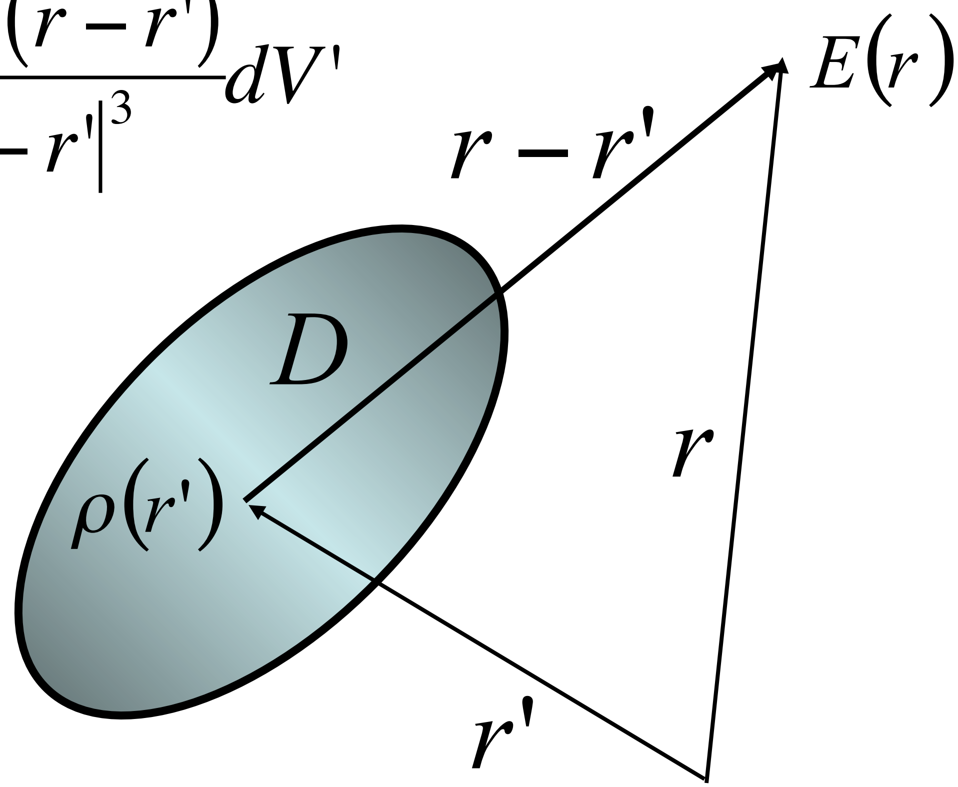
$$F = q\left(E + v \times B\right)$$



ME1: Sources for **E** field

$\nabla \cdot E = \rho$ (**Gauss' Law**)

$$E(r) = -\frac{1}{4\pi}\iiint_D \frac{\rho(r')(r-r')}{|r-r'|^3} dV'$$



A Current induces a Magnetic field

The Biot-Savart Law
(magnetostatics)

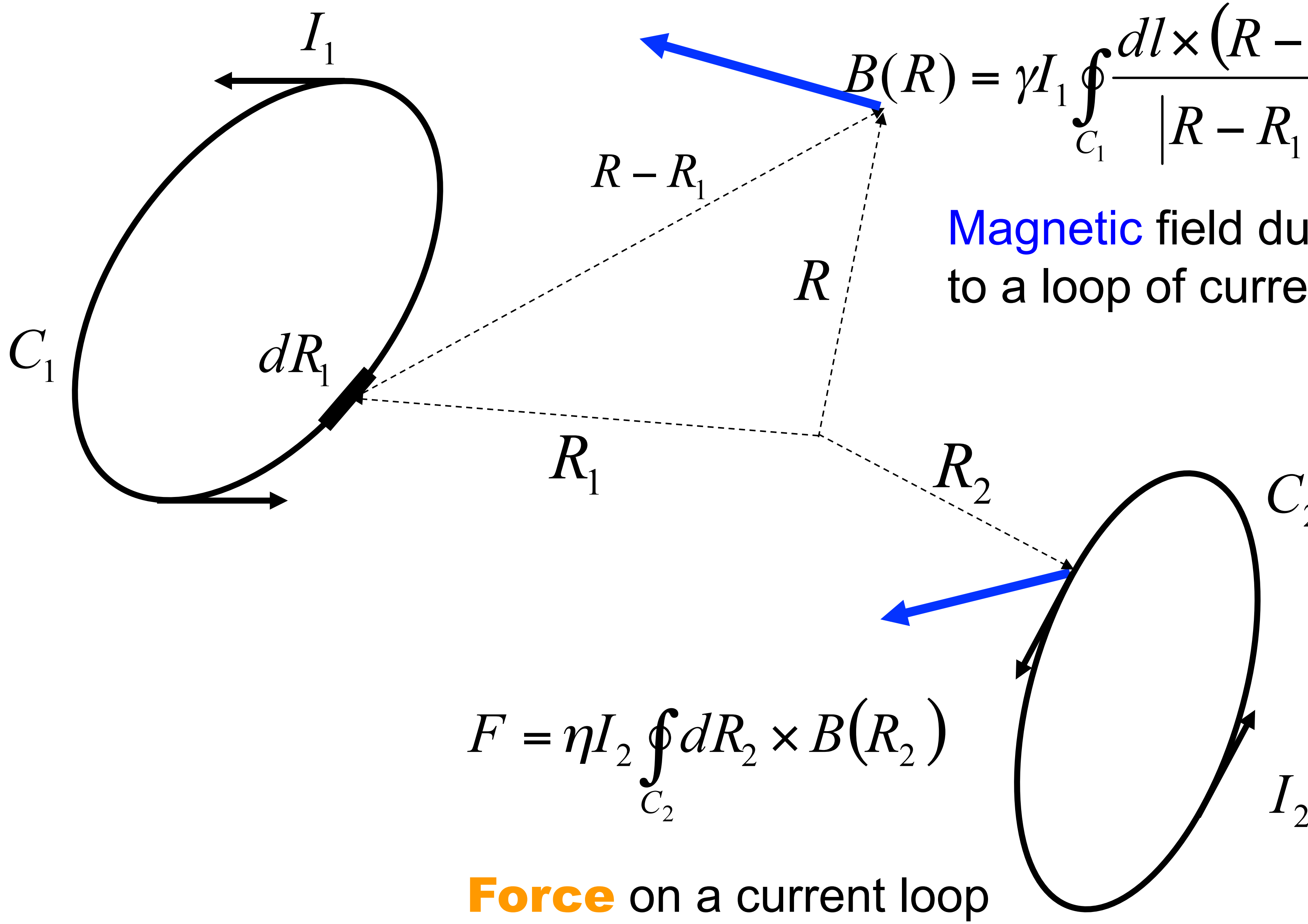
$$\boxed{\nabla \times B = J}$$

$$B(r) = \frac{1}{4\pi} \iiint_V \frac{J(r') \times (r - r')}{|r - r'|^3} \, dV'$$

$$\boxed{\nabla \cdot B = 0}$$

$$B = \nabla \times A$$

ME2: No Magnetic Charges



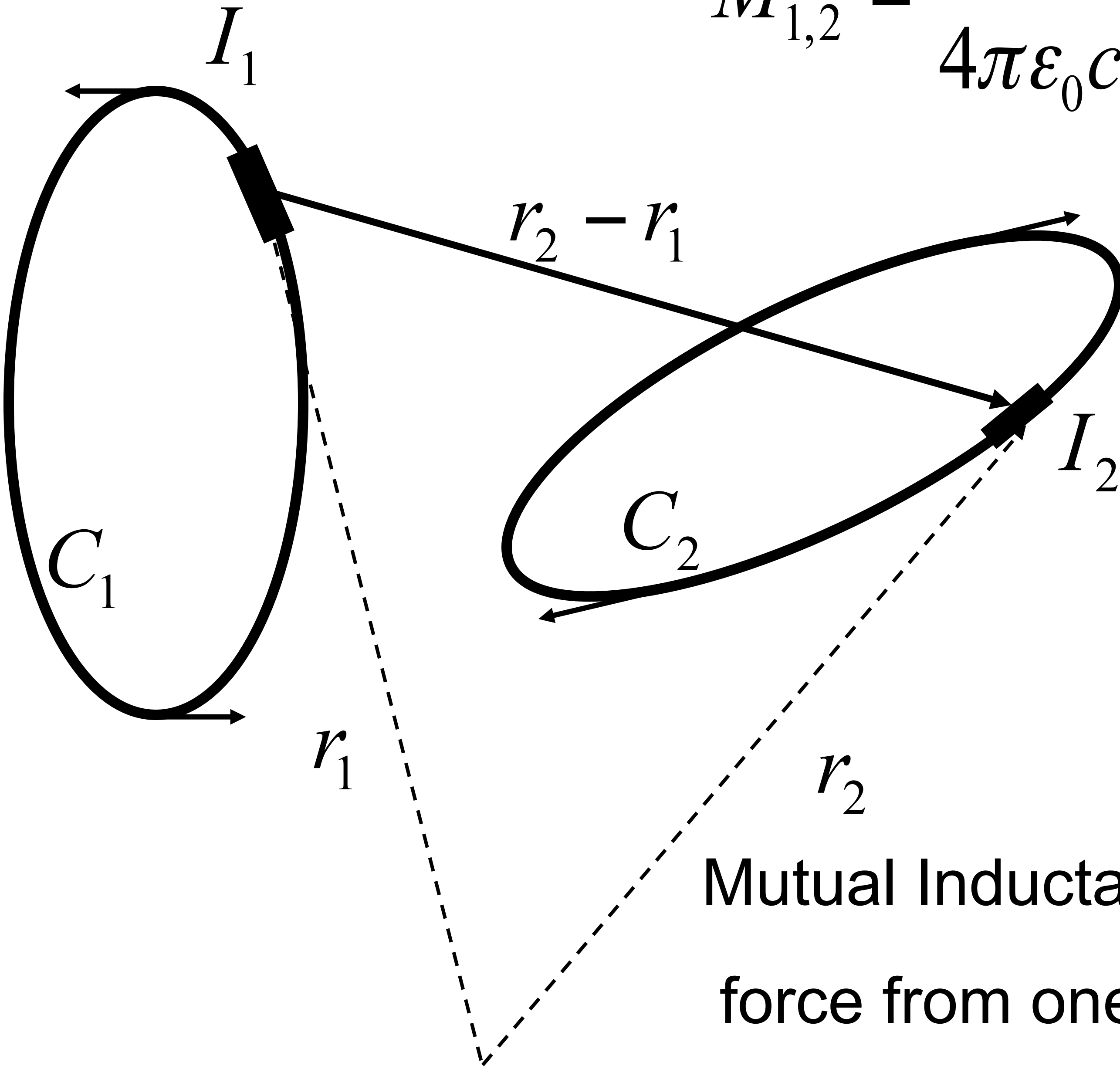
$$B(R) = \gamma I_1 \oint_{C_1} \frac{dl \times (R - R_1)}{|R - R_1|^3}$$

Magnetic field due to a loop of current

$$F = \eta I_2 \oint_{C_2} dR_2 \times B(R_2)$$

Force on a current loop due to a magnetic field

$$M_{1,2} = \frac{1}{4\pi\varepsilon_0 c^2} \oint_{C_1} \oint_{C_2} \frac{dr_1 \cdot dr_2}{|r_1 - r_2|}$$

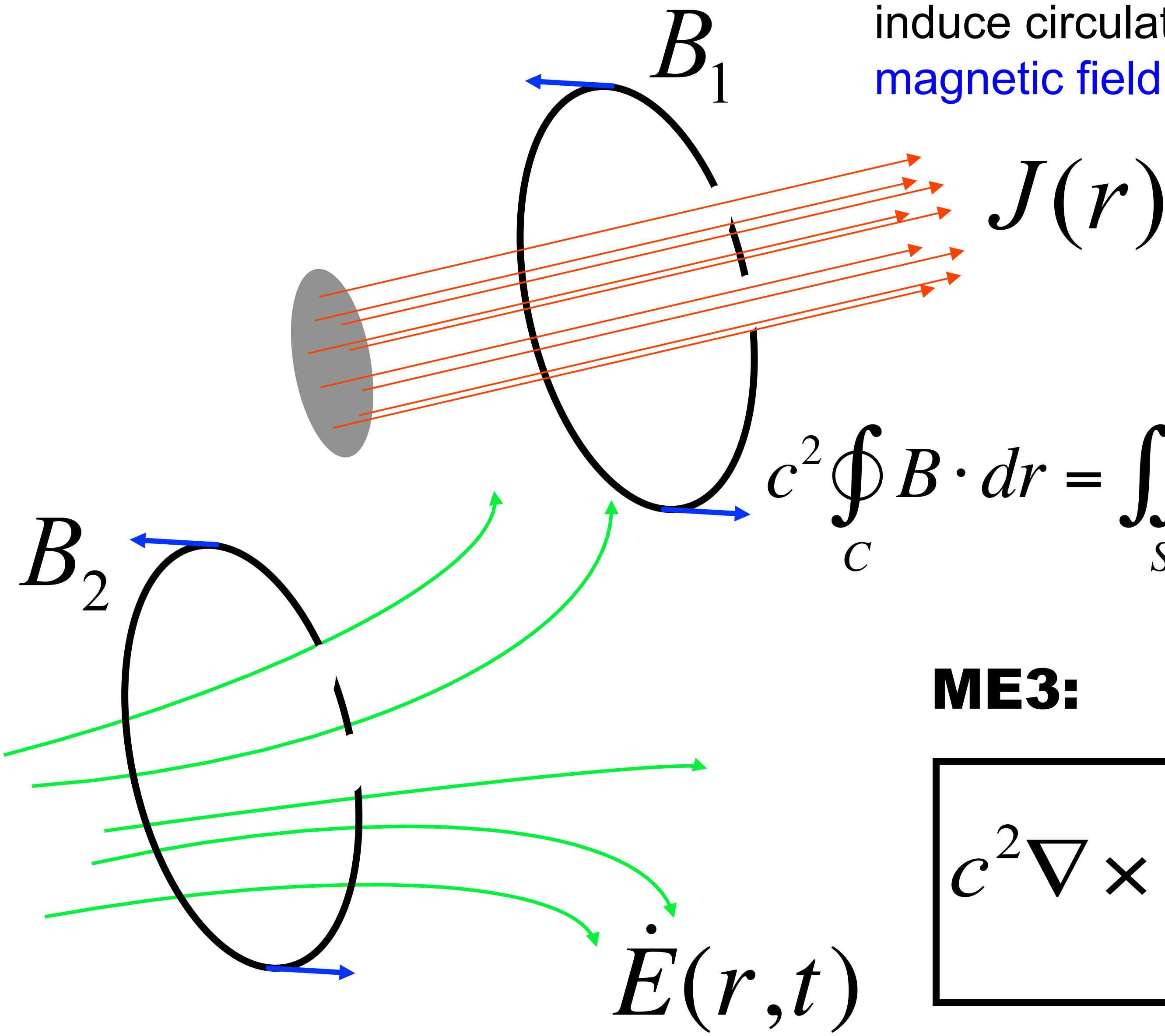


Mutual Inductance of two coils:

force from one coil to another

Ampere's Law

A changing electric flux through a loop or a current flux, both induce circulation of the magnetic field around the loop



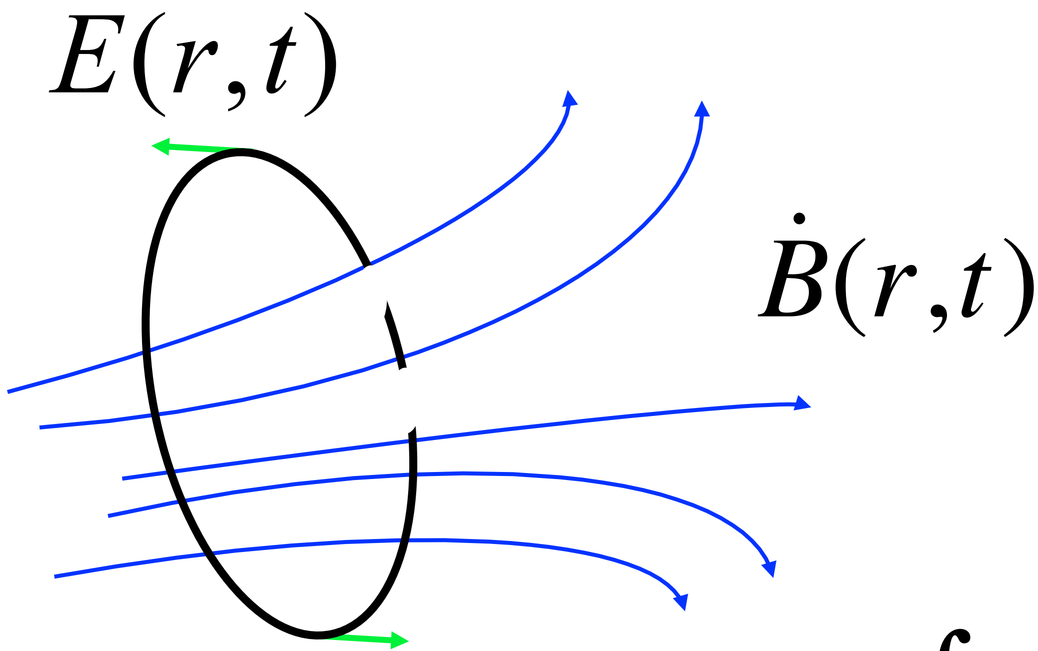
$$c^2 \oint_C B \cdot dr = \iint_S \left(\frac{J}{\varepsilon_0} + \frac{\partial E}{\partial t} \right) \cdot dS$$

ME3:

$$c^2 \nabla \times B = \frac{J}{\varepsilon_0} + \frac{\partial E}{\partial t}$$

Faraday's Law

A changing magnetic flux through the loop, induces a circulation of the electric field around the loop

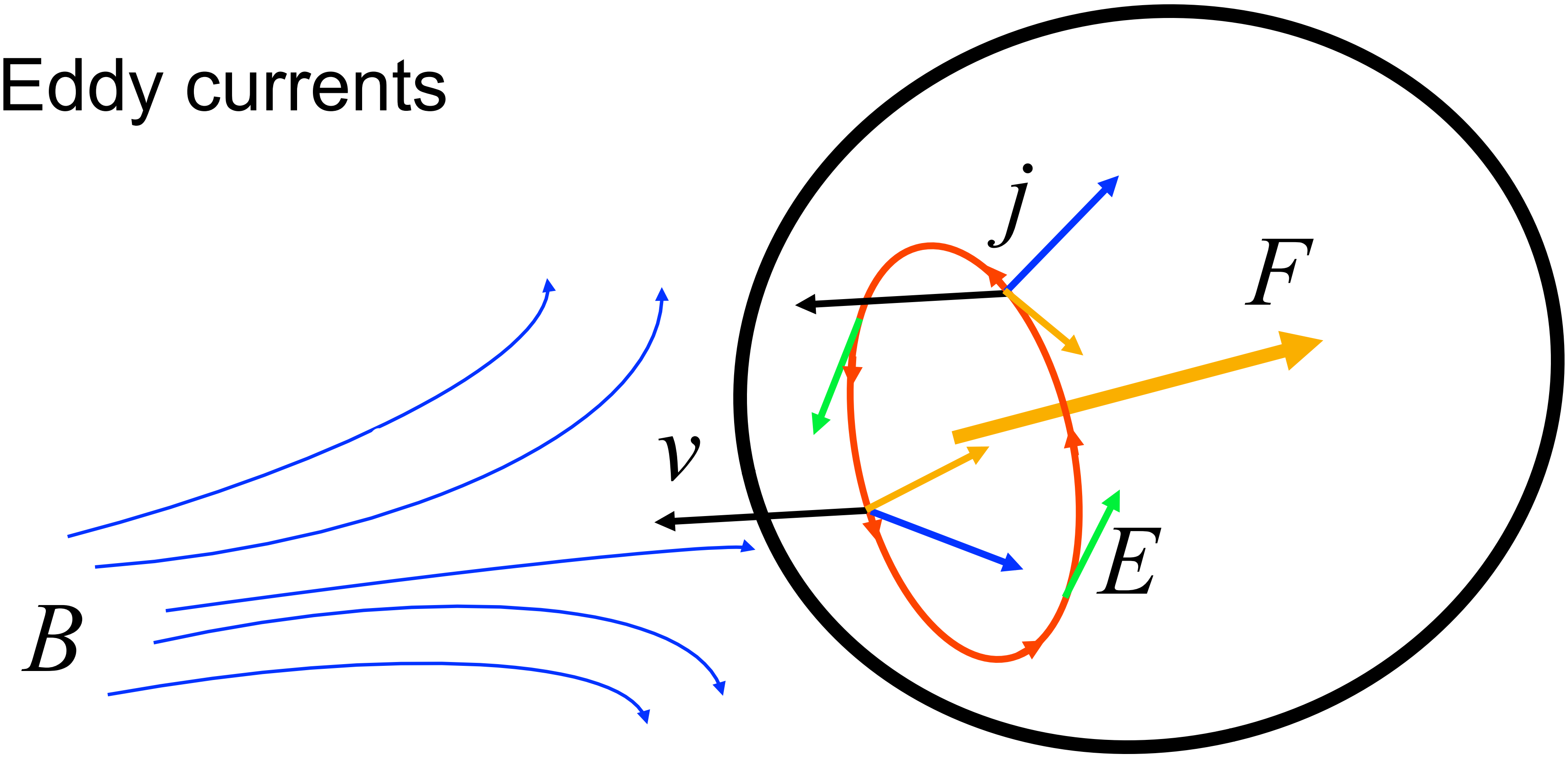


$$\oint_C E \cdot dr = -\iint_S \frac{\partial B}{\partial t} \cdot dS$$

ME4:

$$\boxed{\nabla \times E = -\frac{\partial B}{\partial t}}$$

Eddy currents

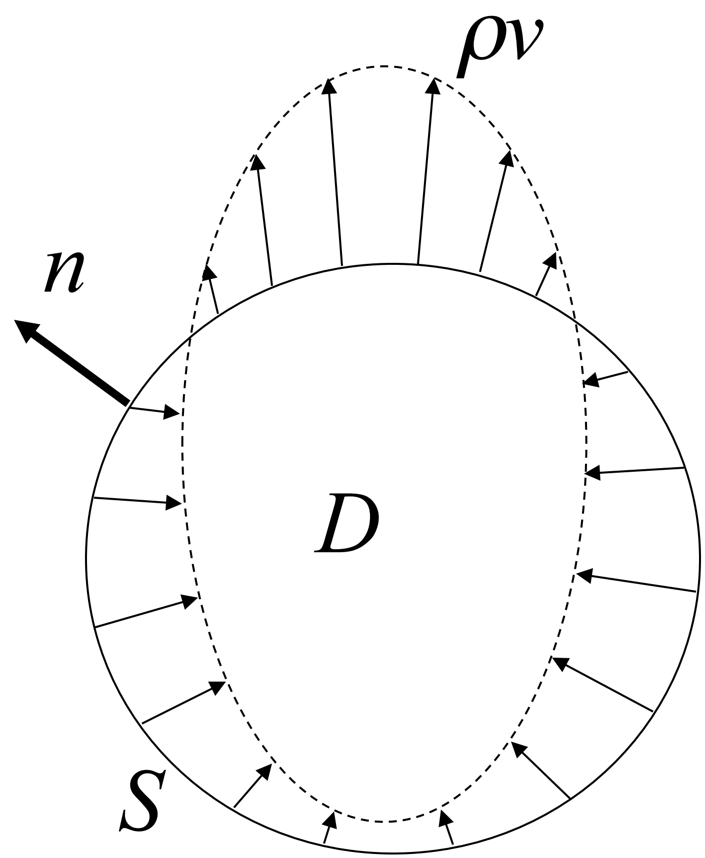


A conducting object moving towards increasing magnetic flux will develop electric field circulation normal to the field lines, which will produce current eddys. These will in turn experience a Lorenz force with a net component opposing the motion: since the magnetic field is irrotational, moving into a region of increasing magnitude means that the field lines cutting the loop are diverging.

The continuity equation

$$\frac{d}{dt}\iiint_D \rho(r,t)\,dV = -\oiint_S \rho v \cdot dS$$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho v) = 0$$



The rate of change of the material (charge) in a control volume **D** equals the net flux into the volume

Electromagnetic waves

$$E = -\nabla\Phi - \frac{\partial A}{\partial t}$$

$$\nabla^2\Phi - \frac{1}{c^2}\frac{\partial^2\Phi}{\partial t^2} = -\frac{\rho}{\varepsilon_0}$$

$$\nabla^2 A - \frac{1}{c^2}\frac{\partial^2 A}{\partial t^2} = -\frac{J}{\varepsilon_0 c^2}$$

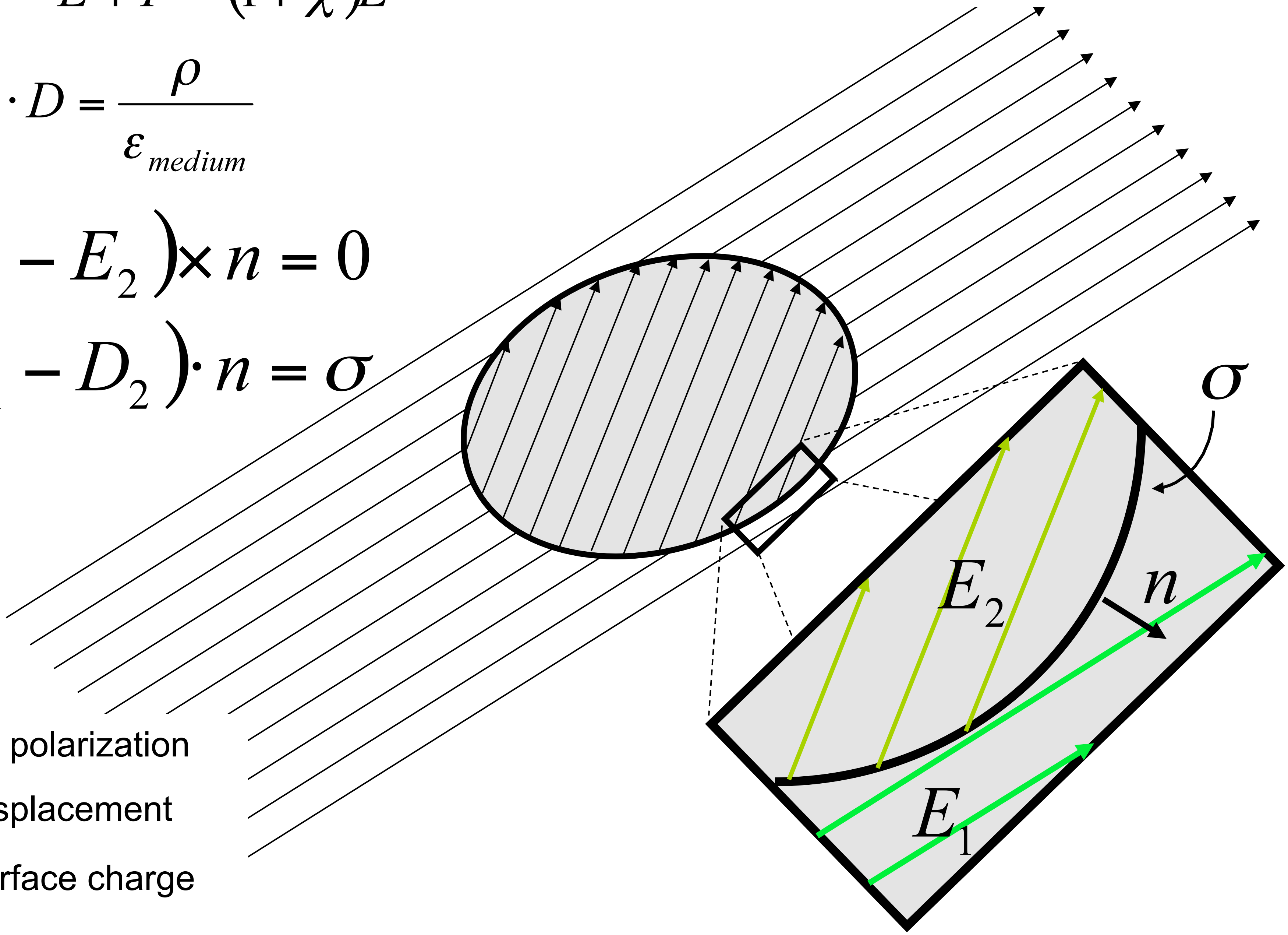
Dielectrics

$$D = E + P = (1 + \chi)E$$

$$\nabla \cdot D = \frac{\rho}{\varepsilon_{medium}}$$

$$(E_1 - E_2) \times n = 0$$

$$(D_1 - D_2) \cdot n = \sigma$$



P(E): polarization

D: displacement

σ: surface charge

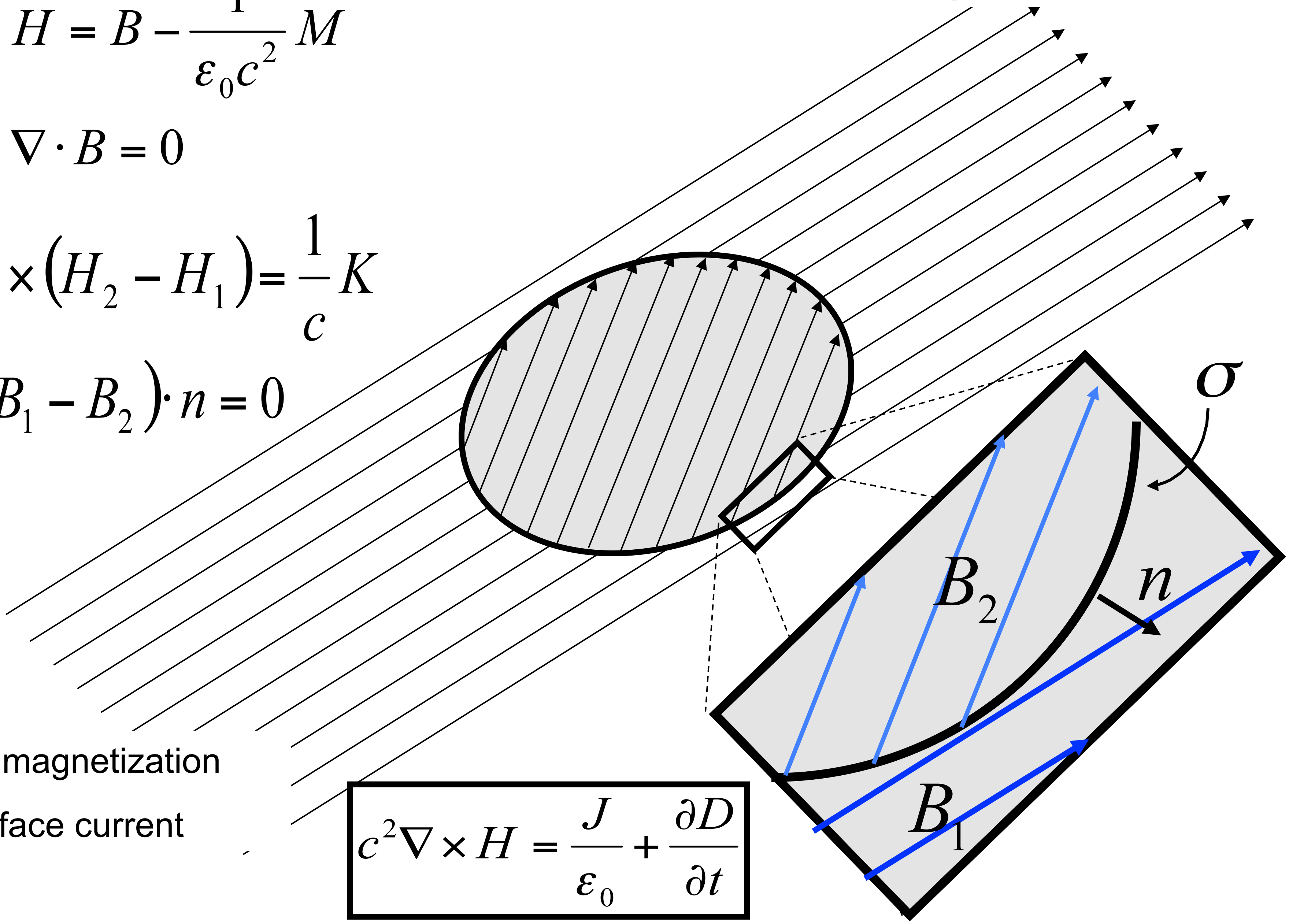
Diamagnetics

$$H = B - \frac{1}{\varepsilon_0 c^2} M$$

$$\nabla \cdot B = 0$$

$$n \times \left(H_2 - H_1\right) = \frac{1}{c} K$$

$$\left(B_1 - B_2\right) \cdot n = 0$$



M(B): magnetization

K: surface current

$$c^2 \nabla \times H = \frac{J}{\varepsilon_0} + \frac{\partial D}{\partial t}$$

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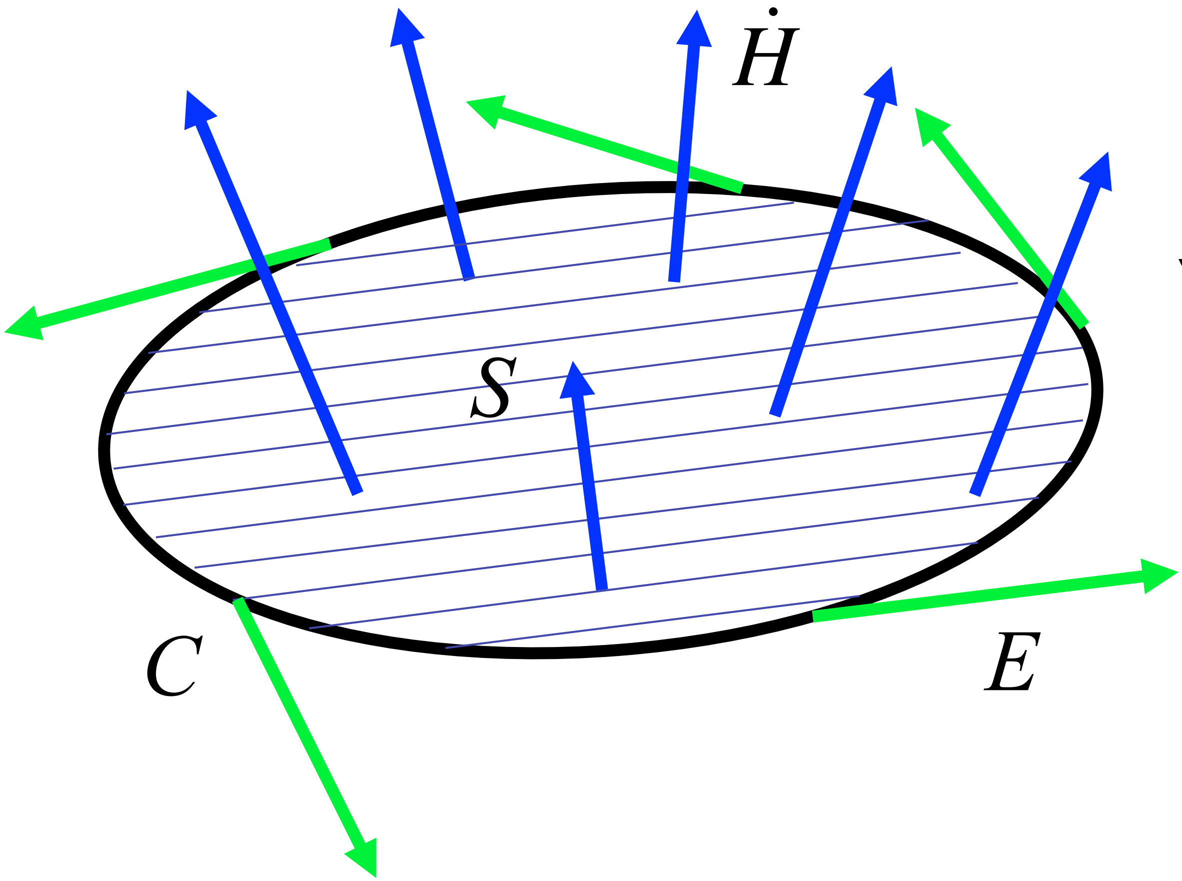
$$\oint_c E \cdot dR = -\frac{1}{c}\partial_t \iint_S H \cdot dS$$



$$\nabla \times E = -\frac{1}{c}\partial_t H$$