102 Test review Problems

Inequalities

1. Solve the inequality for \( x \): \( 3x - 6 \geq \frac{1}{5} x + 2 \)

\[
2(3x - 6) \geq 2\left(\frac{1}{5} x + 2\right) \\
6x - 12 \geq x + 4 \\
5x - 12 \geq 0 \\
\frac{5x}{5} \geq \frac{12}{5} \\
x \geq \frac{12}{5} \text{ Answer}
\]

2. Solve the inequality for \( x \): \( -x - 16 \leq -\frac{3}{4} x + 12 \)

\[
4(-x - 16) \leq 4(-\frac{3}{4} x + 12) \\
-4x - 64 \leq -3x + 48 \\
-x \leq 112 \\
x \geq -112 \text{ Answer}
\]

Simplification of exponents and writing the answer using positive exponents only:

1. Simplify \( \frac{z^2}{z^7} \)

\[z^{2-7} = z^{-5} = \frac{1}{z^5} \text{ Answer}\]

2. Simplify: \( p^2q^{-1}(3p^3q^4)^2 \)

\[3p^2q^{-1}(3p^6q^8)^{-1} = 3p^{2+6}q^{-1+8}s^{-8} \]

\[= 3p^8q^7s^{-8} \text{ Answer}\]

3. Simplify:

\[
\left(x^{-4}z\right)^{-3} \left(\frac{2xu^2}{z^{-1}}\right)^{-3} = x^{-4}z^{-3} -3 -6 \\
\frac{z^{-3} -6}{2xu^{-7}} = \frac{z}{2^3x^7u^6} \text{ Answer}
\]
Simplification of radicals
1. Simplify:

\[
\frac{\sqrt{81}}{\sqrt{25}} = \frac{\sqrt{9^2}}{\sqrt{5^2}} = \frac{9}{5}
\]

Answer:

2. Simplify:

\[
\frac{\sqrt{121}}{\sqrt{36}} = \frac{\sqrt{11^2}}{\sqrt{6^2}} = \frac{11}{6}
\]

Answer:

Factoring problems:

1. 81-125w³

\[
81-125w^3 = (3-5w)(9+15w+25w^2)
\]

Answer:

or 8(x+2)(x²-2x+4)

2. 8x³ + 64

\[
8x^3 + 64 = 2(x^3 + 8)
\]

or 8(x+2)(x²-2x+4)

3. 16y⁴

\[
16y^4 = (4y^2)^2 = (2y^2)(4y^2)
\]

or (2-y)(2+y)(4+y²)

Answer:

4. x² - 4a²

\[
x^2 - 4a^2 = (x-2a)(x+2a)
\]

Answer:

5. 10x² - 6xy - 28y²

\[
10x^2 - 6xy - 28y^2 = 2(5x^2 - 3xy - 14y^2)
\]

or \[2 \left( 5x^2 - 14y^2 \right) \]

Answer:

6. 3x² - 5x - 2

\[
3x^2 - 6x + x - 2 = 3x(x - 2) + 1(x - 2)
\]

or \[ (3x+1)(x-2) \]

Answer:

7. 4x² - 12x - 7

\[
4x^2 + 2x - 14x - 7 = 2x(2x+1) - 7(2x+1)
\]

or \[ (2x-7)(2x+1) \]

Answer:
Solve for the unknown indicated in each case:

1. \((u-1)^2 - 8 = 0\) where \(u\) is a real number

\[
\begin{align*}
(u-1)^2 &= 8 \\
\sqrt{(u-1)^2} &= \pm \sqrt{8} \\
|u-1| &= \pm \sqrt{2^2} \\
\frac{u-1}{1} &= \pm 2 \\
\Rightarrow u &= 1 \pm 2\sqrt{2} \\
\text{Answer: } 1 + 2\sqrt{2}, 1 - 2\sqrt{2}
\end{align*}
\]

2. \((x+4)^2 - 12 = 0\) where \(x\) is a real number

\[
\begin{align*}
(x+4)^2 &= 12 \\
\sqrt{(x+4)^2} &= \pm \sqrt{12} \\
x+4 &= \pm \sqrt{2^2 \cdot 3} = \pm 2\sqrt{3} \\
\Rightarrow x &= -4 \pm 2\sqrt{3} \\
\text{Answer: } -4 + 2\sqrt{3}, -4 - 2\sqrt{3}
\end{align*}
\]

3. Solve: \(3x^2 = 10x - 3\)

\[
\begin{align*}
3x^2 - 10x + 3 &= 0 \\
3x(x-3) - (x-3) &= 0 \\
(x-3)(3x-1) &= 0 \\
x &= 3 \quad \text{or} \quad x = \frac{1}{3}
\end{align*}
\]

4. Solve: \(y^4 - 5y^2 + 4 = 0\)

Let \(y^2 = u\),

\[
\begin{align*}
u^2 - 5u + 4 &= 0 \\
(u-1)(u-4) &= 0 \\
u &= 1 \quad \text{or} \quad u = 4 \\
y^2 &= 1 \quad \text{or} \quad y^2 = 4 \\
y &= \pm 1 \quad \text{or} \quad y = \pm 2
\end{align*}
\]

Parabolas

1. Find the x-intercept(s) and the coordinates of the vertex for the parabola \(x^2 + 8x + 15\)

Set \(x^2 + 8x + 15 = 0\)

\[
(x+3)(x+5) = 0
\]

\[
x = -3 \quad \text{or} \quad x = -5
\]

\[
\text{Vertex: } x = -4, y = (-4)^2 + 8(-4) + 15 = 16 - 32 + 15 = -1
\]

\[
\text{Vertex: } x = (4, -1) \quad \text{Answer: } (4, -1)
\]

2. Graph the parabola \(y = (x-3)^2 + 1\)

\[
\begin{align*}
\text{Vertex: } & (3, 1) \\
\text{Since } x &= 1, 2, 3, 4, 5 \quad \text{for } \text{symmetry}, y = 2
\end{align*}
\]

\[
\begin{align*}
\text{For } x &= 2; \quad y = (2-3)^2 + 1 = 1+1 = 2
\end{align*}
\]

\[
\begin{align*}
\text{For } x &= 1; \quad y = (1-3)^2 + 1 = 4+1 = 5
\end{align*}
\]

\[
\begin{align*}
\text{For } x &= 4 \quad \text{due to symmetry, } y = 2
\end{align*}
\]

\[
\begin{align*}
\text{For } x &= 5 \quad \text{due to symmetry, } y = 5
\end{align*}
\]

\[
\begin{align*}
\text{Chose } x &= 1, 2, 3, 4, 5
\end{align*}
\]
Word Problems

1. A ball is thrown from an initial height of 8 feet with an initial upward velocity of 17 ft/s. The ball's height \( h \) (in feet) after \( t \) seconds is given by the quadratic equation \( h = 8 + 17t - 16t^2 \). Find all values of \( t \) for which the height of the ball is 12 feet. Round the answers to the nearest hundredth.

\[
8 + 17t - 16t^2 = 12
\]

\[
-8 - 17t + 16t^2 = 0
\]

\[
= 48 - 17t + 16t^2 \Rightarrow a = 16, b = -17, c = 84
\]

\[
t = \frac{-(-17) \pm \sqrt{(-17)^2 - 4(16)(4)}}{2(16)} = \frac{17 \pm \sqrt{33}}{32}
\]

\[
t = \frac{0.71 \text{ seconds or } 0.35 \text{ seconds}}{2}
\]

2. A ball is thrown vertically upward. After \( t \) seconds, its height \( h \) (in feet) is given by the function \( h(t) = 32t - 16t^2 \). How long would it be before the ball reaches the maximum height?

\[
a = -16
\]

\[
b = 32
\]

\[
\text{X value of vertex} = \frac{-b}{2a} = \frac{-32}{2(-16)} = \frac{1}{1} \text{ sec Answer}
\]
3. The length of a rectangle is 2 feet more than twice its width. Its area is 40 ft\(^2\). What are its dimensions?

\[
\begin{align*}
L &= 2w + 2 \\
A &= L \cdot W = 40 \\
(2w + 2) \cdot W &= 40 \\
2w^2 + 2w - 40 &= 0 \\
2(w^2 + w - 20) &= 0 \\
2(w + 5)(w - 4) &= 0 \\
w &= -5 \text{ or } w = 4
\end{align*}
\]

Choose:

\[
\begin{align*}
W &= 4 \\
L &= 2w + 2 \\
&= 2(4) + 2 \\
&= 8 + 2 = 10
\end{align*}
\]

\[
\begin{align*}
L &= 10 \text{ feet} \\
W &= 4 \text{ feet}
\end{align*}
\]

Answer:

4. A total of $15,000 is invested between two funds A and B. Fund A pays 5\% interest per year and Fund B pays 7\% interest per year. If the annual interest from both funds is $850, how much money was invested in each fund?

Let Fund A investment = x; Fund B investment = y.

\[
\begin{align*}
x + y &= 15,000 \\
y &= 15,000 - x
\end{align*}
\]

Profit: Interest:

\[
\begin{align*}
0.05x + 0.07y &= 850 \\
0.05x + 0.07(15,000 - x) &= 850 \\
0.05x + 1050 - 0.07x &= 850 \\
-0.02x &= -200 \\
x &= 10,000 \\
y &= 15,000 - 10,000 \\
y &= 5,000
\end{align*}
\]

5. The Sugar Sweet Company is going to transport its sugar to market. It will cost $4000 to rent trucks, and it will cost an additional $200 for each ton of sugar transported. Let represent the total cost (in dollars), and let represent the amount of sugar (in tons) transported. Write an equation relating to and then graph your equation using the axes below.

\[
C = 4000 + 200S
\]

\[
C = \text{cost ($)}
\]

\[
S = \text{tons of sugar transported}
\]

If \( S = 0 \), \( C = 4000 \)

If \( S = 5 \)

\[
\begin{align*}
C &= 4000 + 200(5) \\
&= 4000 + 1000 \\
&= 5000
\end{align*}
\]

Plot \((0, 4000)\) and \((5, 5000)\) on the graph. Join them.