MATH 121 Final Exam Practice Problems for ALEKS-trained students

Multiple Choice Section. Write the letter of your answer in the box provided

1. Solve the equation \( x^2 - x - 2 = 0 \) for \( x \).
   (a) \( x = 2, 1 \) \hspace{1cm} (b) \( x = -2, 1 \) \hspace{1cm} (c) \( x = 2, -1 \) \hspace{1cm} (d) \( x = -2, -1 \) \hspace{1cm} (e) None of the above
   \[
   x^2 - x - 2 = 0 \\
   (x-2)(x+1)=0 \\
   \text{Therefore } x = 2 \text{ or } x = -1 \quad \text{Ans (c)}
   \]

2. Write the solution to the inequality \( |x| < 12 \) using interval notation.
   (a) \((-12, 12)\) \hspace{1cm} (b) \((-\infty, 12)\) \hspace{1cm} (c) \((12, \infty)\) \hspace{1cm} (d) \((-\infty, -12) \cup (12, \infty)\) \hspace{1cm} (e) None of the above
   \[
   \begin{array}{c}
   \includegraphics[scale=0.5]{interval_notation.png} \\
   (-12, 12) \\
   \text{Ans (a)}
   \end{array}
   \]

3. Find the distance between the points \((-1, -3)\) and \((2, 3)\).
   (a) 9 \hspace{1cm} (b) \(\sqrt{36}\) \hspace{1cm} (c) \(\sqrt{45}\) \hspace{1cm} (d) \((-3, 0)\) \hspace{1cm} (e) None of the above
   \[
   \begin{align*}
   (x_1, y_1) &= (-1, -3) \\
   (x_2, y_2) &= (2, 3) \\
   d &= \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2} \\
   &= \sqrt{(2-(-1))^2 + (3-(-3))^2} \\
   &= \sqrt{3^2 + 6^2} = \sqrt{9 + 36} = \sqrt{45} \\
   \text{Answer: (c)}
   \end{align*}
   \]

4. The x-intercepts of \( x^2 + y^2 = 16 \) are
   (a) \((-4, 0), (4, 0)\) \hspace{1cm} (b) \((4, 0)\) \hspace{1cm} (c) \((-16, 0), (16, 0)\) \hspace{1cm} (d) \((0, -4), (0, 4)\)
   \[
   \begin{array}{c}
   \includegraphics[scale=0.5]{circle.png} \\
   \text{Circle with center (0,0) and radius = 4} \\
   \text{Ans (a)}
   \end{array}
   \]
5. Find the slope of the line that is perpendicular to $2x - 5y = 7$.
   \[ \frac{2}{5} \quad (b) \quad \frac{-3}{2} \quad (c) \quad \frac{3}{2} \quad (d) \quad 5 \quad (e) \text{ None of the above.} \]

   \[-2y = -2x + 7 \quad -5y = -2x + 7 \quad y = \frac{-2x}{5} \quad \frac{3}{2} \times \frac{-2}{5} \quad \text{Slope of this line} = \frac{3}{2} \quad \text{Slope of a line perpendicular to it} \quad \text{is} \quad \frac{5}{2} \quad \text{Ans:} \quad (b) \]

6. Find the midpoint of the line segment joining the points \((\frac{7}{2}, 3)\) and \((\frac{-1}{2}, -9)\).
   \[(a) \quad (1, -3) \quad (b) \quad (\frac{2}{3}, 12) \quad (c) \quad \frac{\sqrt{2}}{2} \quad (d) \quad (2, 6) \quad (e) \text{ None of the above} \]

   \[ \text{Midpoint formula} \quad \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \]
   \[ \left( \frac{\frac{7}{2} + \frac{-1}{2}}{2}, \frac{3 + (-9)}{2} \right) \]
   \[ \left( \frac{3}{2}, -\frac{6}{2} \right) \quad \text{Ans:} \quad (a) \]

7. Find the equation of the circle in standard form that has center \((1, -3)\) and radius of \(\sqrt{5}\).
   \[(a) \quad (x + 1)^2 + (y - 3)^2 = \sqrt{5} \quad (b) \quad (x - 1)^2 + (y + 3)^2 = \sqrt{5} \quad (c) \quad (x + 1)^2 + (y - 3)^2 = 5 \quad (d) \quad (x - 1)^2 + (y + 3)^2 = 5 \quad (e) \text{ None of the above} \]

   \[ (x - 1)^2 + (y + 3)^2 = 5 \quad \text{Ans:} \quad (d) \]

8. Find the average rate of change of \(f(x) = 3x^2\) from \(x = 1\) to \(x = 2\).
   \[(a) \quad 4.5 \quad (b) \quad 6 \quad (c) \quad 6 \quad (d) \quad 9 \quad (e) \text{ None of the above} \]

   \[ \text{Average rate of change} = \frac{f(x_2) - f(x_1)}{x_2 - x_1} \]
   \[ = \frac{3^2 - 3^2}{2 - 1} = \frac{9 - 3}{1} = \frac{6}{1} = 6 \quad \text{Ans:} \quad (c) \]

---

Use the given graph of the function \(f\) at the right to answer problems #9-12

9. Determine the absolute maximum value of \(f\).
   \[(a) \quad 3 \quad (b) \quad 4 \quad (c) \quad -2 \quad (d) \quad -1 \quad (e) \text{ None of the above} \]

   \[f(x) = 0 \text{ happens at two locations} \quad (x = 1, x = 5) \]

   \[\text{Absolute maximum value of } f \text{ occurs here} \quad \text{Ans:} \quad (b) \]

   \[\text{Note: Difference between absolute and relative maximum (or minimum)} \]

   \[\text{TIP} \]

   \[\text{In this graph B and D are locations of relative maxima; however, C is the location of the absolute maximum.} \]

10. Find a value of \(x\) for which \(f(x) = 0\).
    \[(a) \quad x = 2 \quad (b) \quad x = 1 \quad (c) \quad x = -2 \quad (d) \quad x = 3 \quad (e) \text{ None of the above} \]

    \[\text{Note: } f(x) = 0 \text{ means } y = 0 \text{ on the graph for the given function.} \]

   \[\text{Ans:} \quad (b) \]

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11. Find the coordinates of the relative (local) maximum.
   (a) (6,1)  (b) (3,0)  (c) (−1,4)  (d) (−3,−3)  (e) None of the above
   Answer: C

   Note: In this case the relative maximum (also called local maximum) is the same as the location of the absolute maximum.

12. Find all of the x-intervals where the graph is increasing.
   (a) (−3,4) U (−2,3)  (b) (3,6)  (c) (−3,−1) U (3,6)  (d) (−1,3)  (e) None of the above
   Answer: C

13. Choose the only function which is one-to-one.
   (a) \( f(x) = x^2 \)  (b) \( f(x) = -3 \)  (c) \( f(x) = 2 - x^2 \)  (d) \( f(x) = e^{-3x} \)
   Answer: Graph (c)

   One-to-one: Use horizontal line test. Graphs (a), (b), and (c) fail the horizontal line test. Graph (d) passes → Answer (d)

14. Determine the vertex of \( f(x) = -2(x-1)^2 - 2 \)
   (a) (−1,−2)  (b) (1,−2)  (c) (−2,−2)  (d) (−2,7)
   Answer: (1,−2)

   Vertex form of a parabola is \( f(x) = a(x-h)^2 + k \)
   Where \((h,k)\) is the vertex.
   In this case \( h = 1, k = -2 \)
   Vertex is at \((1,−2)\)

15. The zeroes of the polynomial function \( f(x) = x^4(x - 3)(x + 1) \) are
   (a) (0,3,−1)  (b) (0,−3,−1)  (c) (4,0,1,−1)  (d) (0,1,1)
   \[ 4x^4(x-3)(x+1) = 0 \]
   \[ x^4 = 0 \]
   \[ x = 0 \]
   Therefore \( x = 0 \) is the solution.

   (b) \( x - 3 = 0 \)  \( x = 3 \)
   Therefore \( x = 3 \) is the solution.

   (c) \( x + 1 = 0 \)  \( x = -1 \)
   Therefore \( x = -1 \) is the solution.

   \( \{0,3,-1\} \) is the answer. Answer: (b)

16. Given that the function \( f(x) = \sqrt{x + 3} \) is one-to-one, what is the range of \( f^{-1}(x) \)?
   (a) Not enough information is given  (b) (−∞,∞)  (c) (−3,∞)
   (d) (0,∞)  (e) [3,∞)
   Answer: (c)

   Range of \( f^{-1}(x) \) = Domain of \( f(x) \)

   To find domain of \( f(x) \)
   Set \( x + 3 \geq 0 \)
   Therefore \( x \geq -3 \) Answer is [−3,∞)

17. The range of \( f(x) = \ln(x-1) \) is
   (a) (1,∞)  (b) (7,∞)  (c) All real numbers
   (d) (−∞,1)  (e) None of the above
   Answer: (d)

   The criterion is \( x - 1 > 0 \) for the domain. The vertical asymptote is at \( x = 1 \).
   At \( x = 1 \), the ln function is undefined as it becomes asymptotic on its way to \( -\infty \).
   Thus the range of the function is (−∞,0).

18. Convert the logarithmic equation \( \log_b x = \frac{1}{2} \) into exponential form.
   (a) \( 4^{1/2} = x \)  (b) \( 2^{1/2} = 4 \)  (c) \( 4^{1/2} = 1/2 \)  (d) \( 2^{1/2} = 4 \)  (e) None of the above
   Logarithmic form \( \log_b x = y \)  \( x = b^y \)
   \( \log_4 x = \frac{1}{2} \) \( x = 4^{1/2} = 2 \) Answer: (a)
19. Find the exact value of the expression: \(e^{2 \ln(3)}\)
(a) 6  (b) \(e^{\ln(10)}\)  (c) 16  (d) 9  (e) None of the above

\[ e^{2 \ln(3)} = e^{\ln(3^2)} = 9 \]

Answer: (d)

20. If \(e^{2}\) is evaluated, the result is:
(a) \(\ln(4)\)  (b) 1.001  (c) negative  (d) positive  (e) None of the above

Exponential functions have only positive values.

Answer: (d)

21. For the circle \(x^2 + (y-1)^2 = 9\) (a) state the center and radius. Graph the circle. Place the circle correctly on the axes. (b) Find the x and y intercepts and label them on your graph.

The standard form of a circle is \((x-h)^2 + (y-k)^2 = r^2\)

With \((h,k)\) being the location of the center and \(r\) the radius of the circle.

Here \((h,k) = (0,1)\)

and \(r = \sqrt{9} = 3\)

22. Find the distance between the two points \((3, -5)\) and \((1, 7)\).

\[ d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2} = \sqrt{(-3)^2 + (-7-3)^2} = \sqrt{9 + 144} = \sqrt{153} \]  Answer

23. Find the midpoint between the two points \((3, 5)\) and \((1, 7)\). All three points on the same grid to verify that the midpoint you found lies between the two given points.

Midpoint = \[\left( \frac{x_1+x_2}{2}, \frac{y_1+y_2}{2} \right) = \left( \frac{3+1}{2}, \frac{5+7}{2} \right) = \left( \frac{4}{2}, \frac{12}{2} \right) = (2, 6)\]

24. Graph \(f(x) = \sqrt{x} - 2\) (a) Label at least two points (b) What is the domain of \(f(x)\) and its range

Left most point is when \(x = 0\)

\(|x| < 2\)  \(x \geq 2\)

25. For the function \(f(x) = x^2 - 2x\), find the average rate of change from \(x = 1\) to \(x = 3\).

\(f(x) = 1 - 2(3) = 1 - 6 = -5\)

26. For the function in problem 25, compute the difference quotient \(\frac{f(x+h) - f(x)}{h}\)

\(f(x) = -2x^2 + 2x - 3\)  \(\text{via factoring}\)

\(f(x) = -2(x^2 - x - \frac{3}{2})\)

\(f(x) = -2(x + \frac{3}{4})^2 - \frac{9}{2} + \frac{3}{2}\)

\(f(x) = -2x^2 + 2x - 3\)  \(\text{difference quotient}\)

\(f(x) = 1 - 2(3) = 1 - 6 = -5\)  \(\text{average change}\)

\(f(x) = 1 - 2(x^2 - x - \frac{3}{2})\)  \(\text{via factoring}\)

\(f(x) = -2(x + \frac{3}{4})^2 - \frac{9}{2} + \frac{3}{2}\)

\(f(x) = 1 - 2x^2 - 2x - \frac{3}{4}\)  \(\text{difference quotient}\)
27. Given the functions \( f(x) = x^2 + 2 \) and \( g(x) = \sqrt{x} - 2 \), find and simplify \((f \circ g)(x)\)

\[ (f \circ g)(x) = \left( \sqrt{x} - 2 \right)^2 + 2 \]

\[ = x - 2 + 2 = 1 \]

Answer.

Just for kicks \((g \circ f)(x) = \sqrt{x^2 + 2} = \sqrt{x^2} = x\)

What does that tell you? \((f \circ g)(x) \neq (g \circ f)(x)\) implies \(f(x)\) and \(g(x)\) are inverses of each other.

28. Given \( y = x^2 + 2x + 3 \)

To find \( \text{vertex} \)

\[ x = \frac{-b}{2a} = \frac{-2}{2(1)} = -1 \]

\[ y = (-1)^2 + 2(-1) + 3 = 1 - 2 + 3 = 2 \]

\[ \text{vertex} = (-1, 2) \]

a. Determine the vertex and axis of symmetry...

\((-1, 2)\)

b. Determine the \(x\) and \(y\) intercepts (if they exist).

**x-intercept:**

\[0 = x^2 + 2x + 3\]

\[x = \frac{-2 \pm \sqrt{(2)^2 - 4(1)(3)}}{2(1)}\]

\[= -2 \pm \sqrt{-8}\]

Cannot be solved

\(\Rightarrow\) no \(x\)-intercept (see graph for why)

**y-intercept:**

\((-3, 0)\)

labelled "C"

29. Given the function \( g(x) = \frac{1}{x^2} + 1 \), determine the following and sketch a graph:

(a) The equation of the vertical asymptote.

\[ VA \hspace{1cm} x = 0 \]

\[ x = -1 \]

(b) The equation of the horizontal asymptote.

Degrees of numerator and denominator are equal.

The vertical \( HA = 1 \)

(c) The \(y\)-intercept(s)

\( (0, \frac{3}{2}) \)

labelled "A"

(d) The \(x\)-intercept(s)

\( (-3, 0) \)

labelled "C"

In (c) note that the \( HA \) has moved up to \( y = 1 \) because \( x \to +1 \) then \( f(x) \to \frac{1}{x^2} + 1 \)

30. Graph the function \( f(x) = e^{-x} - 1 \). State the domain, range, and asymptotes of \( f(x) \).

**Domain:**

\((-\infty, \infty)\)

**Range:**

\((-1, \infty)\)

**Asymptote:**

\( y = -1 \)

Since \( e \approx 2.7 \)

\( y = 2.7 - 1 = 1.7 \)

The second point is approximately \((-1, 1.7)\)

**y-intercept:**

\((0, 0)\)
31. Given that \( x^2 + y^2 + 4x - 5y = 2 \) is the equation of a circle, determine the center and radius (hint: complete the squares).

Center: \((-2, \frac{5}{2})\)
Radius: \(\frac{7}{2}\)

32. Find the inverse of the function: \( f(x) = \frac{2x}{x - 8} \)

\[
\begin{align*}
Y &= \frac{2x}{x - 8} \\
X &= 2y - 8 \\
\text{Cross multiply} &
\end{align*}
\]

Distribute \((y-8)x = 2y \)

\[
\begin{align*}
Y &= 8x \\
X &= x - 2 \\
\text{Therefore} &
\end{align*}
\]

\[
\begin{align*}
f^{-1}(X) &= \frac{8X}{X - 2} \\
\text{Answer} &= X - 2
\end{align*}
\]

33. Solve the equation:

\[
\begin{align*}
8e^{3x} &= 40 \\
e^{3x} &= 5 \\
ln(e^{3x}) &= ln5 \\
3x &= ln5 \\
x &= \frac{ln5}{3} \\
\text{Ans.} &= \frac{ln5}{3}
\end{align*}
\]

34. Solve the equation:

\[
\begin{align*}
\log_2{x-2} + \log_2{x+2} &= 5 \\
\text{Combine} &
\end{align*}
\]

\[
\begin{align*}
\log_2{(x-2)(x+2)} &= 5 \\
\log_2{(x^2-4)} &= 5 \\
\text{Convert to exponential form} &
\end{align*}
\]

\[
\begin{align*}
x^2 - 4 &= 2^5 \\
x^2 &= 32 \\
x &= \pm\sqrt{32} = 6 \text{ or } -6
\end{align*}
\]

35. Graph \( f(x) = \ln(x+2) \). Determine (a) domain, (b) range (c) asymptotes, and (d) x- and y-intercepts.

(a) Domain \((-2, \infty)\)
(b) Range \((-\infty, \infty)\)
(c) Asymptotes \(x = -2\)
(d) Intercepts

\[
\begin{align*}
\text{Verify intercepts} &
\end{align*}
\]

36. Solve \( \log(x-1) + \log(x+2) = 1 \)

Combine \( \log \text{ terms} \)

\[
\begin{align*}
\log(x-1)(x+2) &= 1 \\
\text{Solve} &
\end{align*}
\]

37. Solve \( 3^x = 9 \)

\[
\begin{align*}
\log_3 3^x &= 1 \\
x &= 3 \text{ or } x = -1
\end{align*}
\]

Distribute

\[
\begin{align*}
\log 3^x + \log 3 &= \log 9 \\
\text{Check} &
\end{align*}
\]

38. Solve \( 3 \ln(x-5) = 1 \)

\[
\begin{align*}
\ln(x-5) &= \frac{1}{3} \\
X &= e^{\frac{1}{3}} \\
X &= 5 + e^{\frac{1}{3}} \\
\text{Ans.} &
\end{align*}
\]
39. The function \( f(x) = \frac{3x}{x+2} \) is one-to-one. (a) What is the domain of \( f(x) \) ? (b) What is the range of \( f \)?

(a) \( x+2 \neq 0 \)
\[ x \neq -2 \]
\[ \text{Domain of } f(x) = \mathbb{R} \setminus \{-2\} \]

(b) \( f(9) = \frac{27}{11} \) U \( \{-2\} \) Answer:
\[ f^{-1}(x) = \frac{-2x}{x-3} \]

40. What is \( f^{-1} \) for the function in problem 39 above? What is the domain of the inverse? What is the range of \( f^{-1}(x) \)?

\[ f^{-1}(x) = \frac{-2x}{x-3} \]
\[ \text{Domain of } f^{-1} = (-\infty, 3) \text{ U } (3, \infty) \]
\[ \text{Range of } f^{-1} = \mathbb{R} \setminus \{-2\} \]

41. Moth balls let outside quickly lose their mass due to sublimation into the surrounding air. Suppose that the mass of the moth ball A in ounces varies with time following the equation \( A = A_0 e^{-0.03t} \), where \( A_0 \) is the initial mass in ounces and \( t \) is the time in days.

(a) If we start with 6 ounces of moth balls, how many ounces will be there after 10 days?

\[ A_0 = 6, t = 10 \]
\[ A = 6 e^{-0.03 \times 10} \]
\[ = 6 e^{-0.3} \]
\[ = 6 e^{\ln(0.5)} \]
\[ A = 3 \]

(b) How many days will it take for there to be only 3 ounces of moth balls?

\[ A = 3, t = ? \]
\[ 3 = 6 e^{-0.03t} \]
\[ -0.03t = \ln\left(\frac{1}{2}\right) \]
\[ t = \frac{-\ln(0.5)}{-0.03} \]

42. For the polynomial function \( f(x) = (x-2)^2(x+1) \) (a) what are its zeros and associated multiplicities? (b) Does it cross or touch the x-axis at each of the zeros? (c) Determine the end behavior of the graph (example: does it rise/fall on the left and does it rise/fall on the right?)

(a) \[ f(x) = (x-2)^2(x+1) = 0 \]
\[ x = 2 \text{ (multiplicity 2)} \]
\[ x = -1 \text{ (multiplicity 1)} \]

(b) Graph touches the x-axis at \( x = 2 \) (even multiplicity) and crosses the x-axis at \( x = -1 \) (odd multiplicity).

(c) End behavior: \( f(x) \) is odd (degree 3) with a positive leading coefficient.
\[ \text{Ans: Falls to the left and rises to the right.} \]

43. Graph \( f(x) = \ln(x + 2) \). What is the domain, range, intercepts and asymptotes?

\( VA \) set \( x+2 = 0 \)
\[ x = -2 \]

Graphing:
When \( x = -1, y = 0 \)
When \( x = e^{-2} \approx (0.017), y = 1 \)

Domain: \( \mathbb{R} \) x \( (-\infty, -2) \)

Range: \( \mathbb{R} \) x \( (-\infty, \infty) \)

44. Graph \( f(x) = x^2 - 4x + 3 \). (a) Find the domain and range. (b) What is the vertex and axis of symmetry? (c) What are the x-intercepts? (d) Graph the function and label clearly the vertex and intercepts.

(a) \[ x = \frac{4}{2} = 2 \]
\[ y = (2)^2 - 4(2) + 3 = 3 \]

(b) Vertex: \( x = 2 \)
\[ y = 3 \]

(c) X-intercepts:
\[ (2, -1) \]
\[ (1, 0) \)

(d) Graph the function and label clearly the vertex and intercepts.

45. For the rational function \( f(x) = \frac{2x^3 - 9}{x^2 - 2x - 3} \) (a) What is the domain? (b) Where are the asymptotes?

(a) Domain:
\[ x^2 - 2x - 3 = 0 \]
\[ (x-3)(x+1) = 0 \]
\[ x = 3 \text { or } x = -1 \]
\[ x = 3 \text { is excluded.} \]

So the domain is:
\[ (-\infty, -1) U (-1, 0) U (0, 3) U (3, \infty) \]

(b) VA:
\( x = -1 \)
\( x = 3 \)
\( VA y = 0 \)

Degree of numerator < degree of denominator

**HA y = 0**
46. Graph \( f(x) = (x - 3)^3 \). (a) Label at least two points (b) What is the domain and range (c) Find all intercepts

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
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<tbody>
<tr>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>-1</td>
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<tr>
<td>3</td>
<td>0</td>
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(b) Domain \((-\infty, \infty)\)  
Range \((-\infty, \infty)\)

(c) y-intercept when \(x = 0\)  
\((-3)^3 = -27\)  
Answer  
\(x\)-intercept when \(y = 0\)  
\(x = 3\)  
\((3, 0)\)  
Answer

47. Graph \( f(x) = |x| + 3 \) (a) Label at least three points (b) What is the domain and range (c) Is the function odd, even or neither?

(b) Domain \((-\infty, \infty)\)  
Range \([3, \infty)\)

(c) \( f(-x) = f(x) \)  
Symmetric about the y-axis  
Function is even

49. Given \( f(x) = 2x^2 + 3 \) and \( g(x) = \sqrt{x - 1} \), (a) find \( f \circ g(x) \) and simplify, (b) find \( (f \circ g)(1) \)

(c) What is the domain of \( f \circ g(x) \)

(a) \( (f \circ g)(x) = 2(\sqrt{x-1})^2 + 3 = 2(x - 1) + 3 \)

(b) \( (f \circ g)(1) = 2(1) + 1 = 3 \)  
Answer.

(c) \( (f \circ g)(x) = 2(\sqrt{x-1})^2 + 3 \)

Demand factor \( x - 1 \geq 0 \)  
Or \( x \geq 1 \)  
Answer: \([1, \infty)\)

50. Solve the inequality \( x^2 + 12x > -8x^2 \). Write your answer in interval notation or union of intervals

\[ x^2 + 12x > -8x^2 \]

or \( x^2 + 8x^2 + 12x > 0 \)  
Factor  
\( x (x^2 + 8x + 12) > 0 \)

Factor \( x (x+2)(x+6) > 0 \)  
Positive

Turning points are at \( x = 0, x = -2, x = -6 \)

\[ \begin{array}{|c|c|c|c|c|}
\hline
x & -6 & -2 & 0 & 1 & 2 \\hline
\text{Sign of } x+6 & - & - & + & + & + \\hline
\text{Sign of } x+2 & - & - & + & + & + \\hline
\text{Sign of } x & - & - & + & + & + \\hline
\text{Overall Sign} & - & - & + & + & + \\hline
\end{array} \]

Answer \((-\infty, -2) \cup (0, \infty)\)