Syllabus for "Introductory Numerical Analysis: Numerical Linear Algebra"
Math 504 / CS 575
Spring 2020, University of New Mexico
Version 2, Dated January 22, 2020

Time and Place: Tuesday/Thursday, DSH 126, 12:30pm-1:45pm

Course webpage: http://math.unm.edu/~schroder/2020_Spring_504/index.html

Instructor: Prof. Jacob B. Schroder, jbschroder@unm.edu

Course Credits: 3

Office Hours: SMLC 332

1. Wednesday: 12:30pm-2:00pm
2. Thursday: 2:00pm - 3:30pm
3. By request

Please note that I highly respect parents who are also students. Children are always welcome at my office hours, and I usually have a few toys from my son lying around.

Prerequisites:

1. Math 464 or Math 514
2. We will be programming in numerical Python (can learn along the way)

Text: *Numerical Linear Algebra* by Trefethen and Bau, 1st Edition

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Important Dates:

- Midterm Exam: March 26, 2019
- Spring Break: March 16-20
- **Final Exam: Thursday, May 14, 10:00am-12:00pm** (same room, different time)

Course Description:

This course will introduce you to numerical (or applied) linear algebra. We will cover direct and iterative methods for the solution of linear systems of equations, least squares problems, eigenvalue problems, error analysis, and numerical stability. An introduction to geometric multigrid will take place, time permitting. Assignments will be a mixture of theoretical and programming problems. Programming will be done in numerical Python.

This is a required course for Applied Mathematics Masters students (Plan I and II), and is a standard prep class for the NA Qualifying Exam taken by PhD students.

Schedule of Topics:

- Linear algebra review
  - Basic linear algebra operations, orthogonality, norms, projections
- Singular value decomposition (SVD)
  - Definition, proof of existence, use as a theoretical tool
- QR factorization and projections
  - Gram-Schmidt (modified and unmodified), Householder projections
- Least-squares problems
  - Definition, applications, solving with QR and SVD
- Conditioning and condition numbers
  - Definition, derivation with perturbation theory

- Floating point numbers and arithmetic
    - Impact on numerical computations, leading into stability of algorithms
- Stability
    - Definition, forwards and backwards stability
    - Stability of Householder and back-substitution
- LU factorization (Gaussian Elimination)
    - Pivoting and stability analysis
- Eigenvalue problems
    - Definition, solving through reduction to Hessenberg form
    - Rayleigh quotient iteration, inverse iteration
    - QR iteration with and without shifts
- Iterative methods
    - Overview of basic methods (Jacobi, Gauss-Seidel, SOR)
    - Arnoldi and GMRES
    - CG (conjugate gradients)
- Additional topics may be covered at the instructor's discretion depending on time and student interest.
    - If possible, we will cover an introduction to geometric multigrid

Goals:

- Introduce you to the field of numerical linear algebra, which is really *applied* linear algebra. Broadly speaking, this course should teach you how to effectively work with matrices and vectors in various applied (practical) settings on a computer (i.e., solving linear systems and eigenvalue problems).
- Apply your skills from calculus and linear algebra to derive, design, implement, and test, numerical linear algebra algorithms from the above topics list. Alongside this, you will develop a theoretical understanding of the algorithms, and their behavior

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Learning Outcomes: You will learn how to

- Solve linear systems in a variety of practical settings, including
    - Dense and sparse systems (direct and iterative),
    - Least-squares (solving over- and under-determined systems),
    - Symmetric and nonsymmetric (Cholesky vs. LU and CG vs. GMRES), and
    - Basic iterative methods such as Jacobi, Gauss-Seidel, and SOR.
    - Understand how to choose the appropriate algorithm for a certain problem type.
- Solve eigenvalue problems in a variety of practical settings, including
    - QR iteration, Arnoldi, and Lanczos.
    - Understand how to choose the appropriate algorithm for a certain problem type.
- Learn how to implement the above algorithms on a computer, verify your code, and understand the behavior of your code through the underlying theory.
- Understand the effects of finite precision on numerical computing, including stability, conditioning, and error types.
- Understand the complexity (computational cost) of the covered numerical algorithms.

Grading: The course grade will be determined by

- Homework: 45%
- Midterm Exam: 25%
- Final Exam: 30%

The final grade for the class will be based on the summed weighted percentages above. Letter grades will be assigned as follows:

- A-, 90% or above
- B-, 80% or above
- C-, 70% or above
- D-, 60% or above
- F, below 60%

The instructor reserves the right to curve grades to offset unforeseen circumstances. Such a curve will never decrease a student's letter grade below that from the above scheme.

Absences Policy: It is expected that each student regularly attend class.

- Course follows UNM handbook D-170 https://handbook.unm.edu/d170/
- Note, the handbook says "A student with excessive absences may be dropped from a course by the instructor with a grade of W/P or W/F."
- If you need to miss more than two classes during the semester, please contact the instructor.

Homework: Homework will be posted every 1 to 2 weeks on the course webpage (i.e., here). Each homework will consist of a combination of computer and theoretical problems. You need to hand in a written report on the due date at the start of class. All plots/figures in the report must be generated in Python and not hand drawn (unless otherwise specified in the homework question). If using a Jupyter notebook with text and Latex blocks, a print-out of the notebook may be sufficient to answer some homework problems.

Some guidelines for your homework writeup are

- It should be legibly and neatly presented, and include all the required code, figures and tables, in the order assigned.
- Figures and tables should be well-labeled printouts.
- Any discussion and analysis should be either typed or neatly written.
- All sheets of paper must be stapled, your name written on front.

You are encouraged to work with each other on the homeworks, but the codes and your writeups have to be your own. That is, ***each student must turn in an individual and unique homework writeup.***

Homework may be submitted late up to a week for 50% credit. ***Homework grading disputes must be submitted in writing within one week after the work is returned.***

Software: Use of numerical Python will be required to complete the course homework assignments. Some homeworks may require use of the open-source ***PyAMG*** package. Please see https://github.com/pyamg/pyamg/wiki/Installing
for tips on installing PyAMG.

Exams: There are two exams, a midterm and a final. Cheating on an exam will be handled in accordance with the dishonesty policy below. There are no makeup exams; however, I am sympathetic to a student who is unable to take a scheduled test due to a hardship. Please contact the instructor before the exam (if possible), should such a hardship occur.

Exam grading disputes must be submitted in writing within one week after the work is returned.

Credit Hour Statement: This is a three credit-hour course. Class meets for two 75-minute sessions of direct instruction per week for fifteen weeks during the Fall 2019 semester. Students are expected to complete a minimum of six hours of out-of-class work (or homework, study, assignment completion, and class preparation) each week.

Academic Integrity: Each student is expected to maintain the highest standards of honesty and integrity in academic and professional matters. The University reserves the right to take disciplinary action, including dismissal, against any student who is found responsible for academic dishonesty or otherwise fails to meet the standards. Any student judged to have engaged in academic dishonesty in course work may receive a reduced or failing grade for the work in question and/or for the course.

Academic dishonesty includes, but is not limited to, dishonesty on quizzes, tests or assignments; claiming credit for work not done or done by others; and hindering the academic work of other students; misrepresenting academic or professional qualifications within or without the University; and nondisclosure or misrepresentation in filling out applications or other University records.

Accommodation Statement and Americans with Disabilities Act: In accordance with University Policy 2310 and the Americans with Disabilities Act (ADA), academic accommodations may be made for any student who notifies the instructor of the need for an accommodation. It is imperative that you take the initiative to bring such needs to the instructor's attention, as I am not legally permitted to inquire. Students who may require assistance in emergency evacuations should contact the instructor as to the most appropriate procedures to follow. Contact Accessibility Resource Center at 277-3506 or arc.unm.edu for additional information.

If you need an accommodation based on how course requirements interact with the impact of a disability, you should contact me to arrange an appointment as soon as possible. At the appointment, we can discuss the course format and requirements, anticipate the need for adjustments and explore potential accommodations. I rely on the Disability Services Office for assistance in developing strategies and verifying accommodation needs. If you have not previously contacted them I encourage you to do so.

Title IX: In an effort to meet obligations under Title IX, UNM faculty, Teaching Assistants (TAs), and Graduate Assistants (GAs) are considered "responsible employees" by the Department of Education (see page 15, http://www2.ed.gov/about/offices/list/ocr/docs/qa-201404-title-ix.pdf). This designation requires that any report of gender discrimination which includes sexual harassment, sexual misconduct and sexual violence made to a faculty member, TA, or GA must be reported to the Title IX Coordinator at the Office of Equal Opportunity (http://oeo.unm.edu). For more information on the campus policy (2000, 2740) regarding sexual misconduct, see: https://policy.unm.edu/university-policies/2000/2740.html

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Citizenship and/or Immigration Status: All students are welcome in this class regardless of citizenship, residency, or immigration status. Your professor will respect your privacy if you choose to disclose your status. As for all students in the class, family emergency-related absences are normally excused with reasonable notice to the professor, as noted in the attendance guidelines above. UNM as an institution has made a core commitment to the success of all our students, including members of our undocumented community. The Administration's welcome is found on our website: http://undocumented.unm.edu/

Disclaimer: I reserve the right to make reasonable and necessary changes to the policies outlined in this syllabus. Whenever possible, the class will be notified in advance of such changes. An up-to-date copy of the syllabus can always be found on the course website. It is your responsibility to know and understand the policies discussed therein and to be up-to-date. If in doubt, please ask questions.