

NONLINEAR DYNAMICS AND CHAOS

TOPICS COVERED

1. Introduction

Dynamics: how things change in time
Differential equations vs. difference equations
ODE vs PDE, autonomous coupled systems of ODE's
Nonlinear
Linearity and the Superposition Principle
Linearization of nonlinear problems
Chaos
Long-time behaviour in 1D, 2D, 3D (brief survey)

2. Flows in 1D

Exact solutions: separation of variables
Phase Portrait: fixed points and stability
Linear stability analysis
Existence and Uniqueness
Potential and Kinetic Energy; the limit of infinite damping

3. Bifurcations in 1D

Saddle-Node (or turning point or blue-sky) Bifurcation
Transcritical Bifurcation
Subcritical and Supercritical Pitchfork Bifurcation
Imperfections and Catastrophe (example: budworm population)
Nondimensionalization

4. Flows on the circle (periodic 1D)

Plotting phase portrait on the circle
One and two uniform oscillators
Bifurcations on the circle (illustrated by overdamped pendulum with torque)

5. Flows in 2D : Linear systems

Examples: nodes, saddles, centers
Solutions to $\dot{\mathbf{x}} = \mathbf{A}\mathbf{x}$ of the form $\mathbf{x} = \mathbf{e}^{\lambda t}\mathbf{v}$ (and brief review of 2D matrix theory)
Real Eigenvalues: Stable or Unstable Nodes, Saddles
Complex Eigenvalues: Centers, Stable or Unstable Spirals
Repeated Eigenvalues: Degenerate Nodes, Star nodes

6. Flows in 2D : Nonlinear systems

Linearization near equilibria
Classify linearized system
When does linearized system predict nonlinear behaviour?
Plot Phase Portraits using
Nullclines
Local behaviour near equilibria
Conservative systems
Energy contours and trajectories.
Attracting and repelling fixed points: are they possible? Why not?
Effect of a nonconstant conserved quantity
Pendulum: no damping, underdamped, critically damped, overdamped.

7. Limit Cycles (a nonlinear phenomenon)

Limit Cycles

Significance of limit cycles vs centers

Examples using polar coordinates

Van der Pool Equation

Ruling out Closed Orbits

Linear systems

Conservative systems

Gradient systems, Liapunov functions

Poincare-Bendixson Theorem

Determines everything that can possibly happen in 2D!

Finding trapping regions

8. Bifurcations in 2D

Hopf Bifurcations

Supercritical, subcritical, degenerate

Examples in polar coordinates

Saddle-node Bifurcations

Transcritical and Pitchfork Bifurcations

9. Lorenz System

Properties

Volume contraction

Bounded trajectories

No stable Equilibria if $r > r_H$

No stable Limit cycles if $r > r_H$

Sensitive dependence on initial conditions

Definition of chaos

Definition of an attractor

10. 1-D Maps

Cobwebs

Fixed Points and Stability

Tent map

Boundedness and stretching

No stable orbits

Unstable p-orbits for all p

Logistic map

Fixed points and stability

Flip bifurcation: when p-orbit becomes unstable a stable 2p-orbit appears

Transverse bifurcation: responsible for periodic windows

No stable orbit at finite r

Universality: ratio of lengths l_k/l_{k+1} , order of periodic windows

11. Fractals

Cardinality: countable vs uncountable

Measure: measure of countable sets

Dimension: Similarity, Box, Hausdorff, and Correlation dimension

Ex: Snowflake, Cantor set, randomized Cantor set, Brownian motion, Lorenz attractor

12. Strange Attractors

Attractor Reconstruction