MATH 412 — HOMEWORK # 9 Due : Friday, April 17, 2009.

TOPICS COVERED

Limit Cycles
- Significance of limit cycles vs centers
- Examples using polar coordinates
- Van der Pool Equation

Ruling out Closed Orbits
- Linear systems
- Conservative systems
- Gradient systems, Liapunov functions

Poincare-Bendixson Theorem
- Determines everything that can possibly happen in 2D!
- Finding trapping regions

Reading : Strogatz, Sections 7.0-7.3

1. (a,b) Problems 7.1.1, 7.1.3
2. Problem 7.1.5
3. Problem 7.1.7
4. (a,b) Problem 7.2.1, 7.2.2.
5. Problem 7.2.8 (Hint: Let $(x(t), y(t))$ be a parametrization of an equipotential curve. That is, this is not a solution curve but a curve along which $V(x(t), y(t))$ is constant. Find a vector $\mathbf{t}$ that is tangent to this equipotential curve. You want to show that $\mathbf{t}$ and ∇V are perpendicular to each other. Use the equation $V(x(t), y(t)) = c$.)
6. Problem 7.2.9
7. Problem 7.2.10
8. Problem 7.3.3 (Hint: Show that the trajectories leave small circles but enter large circles, forming a trapping region that contains no equilibria.)