MATH 163 - Final Review - Some solutions

Miscellaneous Series

M1. If a series converges you can approximate it by a finite partial sum. The more terms you include in the partial sum, the closer you get to the limiting value. \( \lim_{n \to \infty} E_n = 0. \)

M2. First find the Taylor series for \( \cosh x \), then note that it converges, and consists of only strictly positive terms, if \( x \neq 0 \). Therefore \( \cosh x \) is strictly bigger than any subset of those terms. 1 + \( x^2/2 \) are the first two terms. If \( x = 0 \), then you get equality.

M4. \((-\infty, \infty)\)

Fundamentals

2. Find derivatives
   (c) \( (1 + 1/x^2)/2 \)  
   (d) \( \tan^{-1}(\sqrt{x})/(2\sqrt{x}) \)  
   (e) \( \sin^{-1}(x^3) + 3x^3/\sqrt{1-x^6} \)

Integration

3. Evaluate definite and indefinite integrals.
   (a) \(-1/x + \ln|x| - x + C \)  
   (b) \((32/25)x^2/\ln(32/25) + C \)  
   (c) \(2\tan^{-1}(x/2) + 1/2 \log(x^2 + 4) - \log|2 - x| \)  
   (d) \(\pi/4 \)  
   (e) diverges  
   (f) diverges \( (p = 1/2) \)  
   (g) 0.01  
   (h) \(\pi/2 \)  
   (i) \(-(t/2) \cos(2t) + (1/4) \sin(2t) + C \)  
   (j) \(e^\theta(\sin \theta - \cos \theta)/2 + C \)  
   (k) \(6/e^5 \)  
   (m)\(2/15 \)  
   (n) \(\tanh^2 x/2 \)  
   (o) \(\tan^{-1}(\sin x) + C \)  
   (p) \((1/8) \int_0^{\pi/4} \cos^2 \theta \, d\theta = (\pi + 2)/41 - (3 \ln |1 - x| + \ln |1 + x|)/2 + C \)

Differential Equations

4. Verify that a given function solves a differential equation: differentiate function and check that it satisfies differential equation. if applicable, also check that it satisfies initial condition.

6. Find the set of all solutions \( y(t) \) to the differential equations
   (a) \( y = t^2/2 + C \)  
   (b) \( y = Ce^t \)  
   (c) \( y = \sin(\sqrt{k}x), y = \cos(\sqrt{k}x) \)

9. Use the method of separation of variables
   (a) \( y(t) = -1/(t + C) \). Missing solution \( y = 0 \). You lose it as soon as you divide by \( y^2 \) to separate variables.

Series

12. Suppose \( \sum_{n=4}^{\infty} a_n = 3 \). \( \lim_{n \to \infty} a_n = 0 \), \( \lim_{n \to \infty} s_n = 3 \).

13. (a) What does it mean for a series to converge? partial sums \( s_n \) approach a finite limit as \( n \to \infty \). this limit is the value of the series.
(b) A series \( \sum_{n=n_0}^{\infty} a_n \) converges if the summands \( a_n \) approach zero sufficiently fast! How fast is fast enough? faster than \( 1/n^p, \ p > 1 \) is sufficient (but not necessary! counterexample?)

16. State a formula for
(a) \( S = \sum_{k=0}^{n} r^k = (1 - r^{n+1})/(1 - r) \)
(b) \( S = \sum_{k=0}^{\infty} r^k = 1/(1 - r), \ |r| < 1. \)

18. Determine whether the following series converge absolutely, conditionally, or diverge. If convergent, can you evaluate?
(a) diverges  
(b) convergent, dont know value  
(c) convergent, = 15/4  
(d) convergent, dont know value  
(e) convergent, = e^{-3}  
(f) convergent, = e^{-3} - 1

19. Consider the power series \( f(x) = \sum_{n=1}^{\infty} \frac{x^n}{n^2} \).
(a) \([-1, 1] \).  
(b) For \( f' \): IC= \([-1, 1] \). For \( f'' \): IC= \((-1, 1) \).