Here is a selection of problems from the 3 reviews and a few on material after exam 3. Please make sure to also understand the problems in the last two homework sets after exam 3.

1. Know graphs (including definitions, domains, intercepts, limiting behaviour) of basic functions $\sin^{-1}(x)$, $\cos^{-1}(x)$, $\tan^{-1}(x)$, $\sinh(x)$, $\cosh(x)$, $\tanh(x)$, $\ln(x)$, $e^x$.

2. Find the derivatives of the following functions
   
   (a) $f(x) = x^{2^x}$  
   (b) $f(x) = \frac{\tan^{-1}x}{x}$  
   (c) $f(x) = \sinh(x)$  
   (d) $f(x) = \int_1\sqrt{x} \tan^{-1} s \, ds$

3. Evaluate the following definite and indefinite integrals. Some of the definite ones may be improper, careful.
   
   (a) $\int \frac{1 + x - x^2}{x^2} \, dx$  
   (b) $\int \frac{x^3}{1 + x^4} \, dx$  
   (c) $\int \frac{x^3}{1 - x^2} \, dx$

   (d) $\int_0^1 \frac{1}{\sqrt{1 - x^2}} \, dx$  
   (e) $\int_0^t (t - s)^2 \, ds$  
   (f) $\int \tan x \, \text{sech}^2 x \, dx$

   (g) $\int \frac{\cos x}{1 + \sin^2 x} \, dx$  
   (h) $\int e^\sqrt{x} \, dx$  
   (i) $\int \frac{1}{2 - 3s} \, ds$

   (j) $\int \tan^{-1} x \, dx$  
   (k) $\int \frac{x^{5x}}{52x} \, dx$  
   (l) $\int_0^1 ve^v \, dv$

   (m) $\int_0^1 \frac{x}{\sqrt{1 - x}} \, dx$  
   (n) $\int_0^1 \frac{1 + x}{1 - x} \, dx$  
   (o) $\int_0^1 \frac{1 + 2x}{1 - x^2} \, dx$

   (p) $\int \frac{dx}{\sqrt{8 - 2x - x^2}}$  
   (q) $\int \frac{16 \cos(x)}{\sin^2(x) - 5 \sin(x) + 6} \, dx$  
   (r) $\int_2^\infty x^2 \, dx$

   (s) $\int_{100}^\infty \frac{1}{x} \, dx$  
   (t) $\int_{-\infty}^\infty e^{-x} \, dx$  
   (u) $\int_{-1}^1 \frac{1}{x} \, dx$

   (v) $\int e^{x^2} \frac{\ln(x)}{x} \, dx$

4. (a) State the Mean Value Theorem. Use it to bound the error in the approximation $e^x \approx 1$ for $|x| < 1$. (That is, find an upper bound for $|e^x - 1|$.)
   
   (b) State the Taylor Remainder Theorem. Use it to bound the error in the approximation $e^x \approx 1 + x + x^2/2$ for $|x| < 1$.

5. Suppose you know $|f'(\xi)| \leq 8$ for all $\xi$ and $f(0) = 0$. Can you give an upper bound for $|f(x)|$ if $x \in [0, 4]$? That is, can you give a number $M$ such that $|f(x)| \leq M$ for all $x$ in the given interval?

6. Find the volume of the torus in homework for Day 14, #2.

7. (a) Use calculus to find the volume of a sphere of radius $R$.
   
   (b) Find the volume of the sphere after a hole of radius $R/2$ has been drilled through its center.
   
   (c) What percentage of the volume of the whole sphere is left after drilling the hole?
8. A heavy uniform cable is used to lift a 300 lb load from ground level to the top of a 100 ft tall building. If the cable weighs 20 lb per linear foot, how much work is done?

9. A filled reservoir is in the shape of a cylinder whose radius and depth are both 100 ft. A pump that floats on the surface of the reservoir pumps water to the top, at which point the water runs off. How much work is done in pumping the water in the reservoir to a depth of 50 ft?

10. Suppose \( \sum_{n=1}^{\infty} a_n = 3 \). Write a formula for the partial sum \( s_n \) of the series. What is \( \lim_{n \to \infty} a_n \)? What is \( \lim_{n \to \infty} s_n \)?

11. Suppose \( \sum_{n=1}^{\infty} a_n 3^n \) converges and \( \sum_{n=1}^{\infty} a_n 6^n \) diverges. What can you say about
   (a) the series \( \sum_{n=1}^{\infty} a_n 2^n \)?
   (b) the series \( \sum_{n=1}^{\infty} a_n \)?
   (c) the series \( \sum_{n=1}^{\infty} a_n 8^n \)?
   (d) the radius of convergence of the power series \( \sum_{n=1}^{\infty} a_n (x-a)^n \)?

12. Determine whether the following series converge absolutely, conditionally, or diverge. If it converges, can you find its value?
   
   (a) \( \sum_{n=1}^{\infty} 1 + n^2 \) 
   (b) \( \sum_{n=1}^{\infty} \frac{2^{2n+1}}{5^n} \) 
   (c) \( \sum_{n=1}^{\infty} \frac{(-1)^n n!}{3^n} \) 
   (d) \( \sum_{n=1}^{\infty} (-1)^n \frac{\sqrt{n}}{n + 1} \) 
   (e) \( \sum_{n=1}^{\infty} \frac{\sin n}{1 + n^2} \) 
   (f) \( \sum_{n=0}^{\infty} e^{-2n} \) 
   (g) \( \sum_{n=1}^{\infty} e^{-2n} \) 
   (h) \( \sum_{n=0}^{\infty} (-1)^n \frac{3^n}{n!} \) 
   (i) \( \sum_{n=1}^{\infty} (-1)^n \frac{3^n}{n!} \) 
   (j) \( \sum_{n=2}^{\infty} \frac{-2}{n(n+1)} \) 
   (k) \( \sum_{n=1}^{\infty} (-1)^n \ln(2 + \frac{1}{n}) \) 
   (l) \( \sum_{n=1}^{\infty} [\ln(n+1) - \ln n] \) 
   (m) \( \sum_{n=1}^{\infty} \frac{1}{\sqrt{n+1}} \) 
   (n) \( \sum_{n=0}^{\infty} (-1)^n \frac{3^{2n+1}}{(2n+1)!} \) 
   (o) \( \sum_{n=2}^{\infty} \frac{1}{n^2 - 1} \) 
   (p) \( \sum_{n=1}^{\infty} \ln n \) 
   (q) \( \sum_{n=2}^{\infty} \frac{1}{n(n\ln n)^2} \) 

13. Let \( f(x) = \sum_{n=1}^{\infty} \frac{x^n}{n^2} \). Find series representations for \( f' \), \( f'' \).

   Find the intervals of convergence of \( f \), \( f' \), and \( f'' \).

14. Determine the radius and the interval of convergence of the following power series.
   
   (a) \( \sum_{n=0}^{\infty} (-1)^n 4^n x^n \) 
   (b) \( \sum_{n=0}^{\infty} \frac{(-x)^n}{n+1} \) 
   (c) \( \sum_{n=1}^{\infty} \frac{3^n(x+4)^n}{\sqrt{n}} \)

15. Find the Maclaurin series for the following functions and state their radius and interval of convergence.
   
   (a) \( f(x) = \frac{2}{3-x} \) 
   (b) \( f(t) = \frac{t}{2t^2 + 1} \) 
   (c) \( f(x) = xe^x \)

   (d) \( f(x) = \tan^{-1}(x) \) 
   (e) \( f(x) = \frac{x}{(2-x)^2} \) 
   (f) \( f(x) = \int_0^x \sin(t^4) \, dt \)
16. Show that \( \pi - \pi^3/3! + \pi^5/5! - \pi^7/7! + \ldots \) converges to zero. How many terms must be computed to get within 0.01 of zero?

17. Estimate the value of \( \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} \) to within an error of 0.001.

18. (a) Find the first 5 terms of the Taylor series for \( f(x) = \sqrt{1+x} \) about \( a = 0 \).
    (b) Use your result in (a) to solve p.783, Problem 72.

19. Use the Maclaurin series for \( \ln(1+x) \) and \( \ln(1-x) \) to show that

\[
\frac{1}{2} \ln \left( \frac{1+x}{1-x} \right) = x + \frac{x^3}{3} + \frac{x^5}{5} + \ldots
\]

20. Let \( F(x) = \int_0^x \frac{\sin t}{t} \, dt \). Show that

\[
F(x) = x - \frac{x^3}{3 \cdot 3!} + \frac{x^5}{5 \cdot 5!} - \frac{x^7}{7 \cdot 7!} + \ldots
\]

   Evaluate \( F(1) \) to three decimal places.

21. (a) Write down the linear approximation of a function \( f(x) \).
    (b) Write down the linear approximation of the change of a function \( f(x) \).
    (b) Use linear approximations to estimate the amount of paint needed to apply a coat of paint 0.05cm thick to a hemispherical dome with diameter 50 m.