TOPICS

23. Power series representation of functions (ctd)

PROBLEMS

DAY 33: Representing functions as series.

1. The function $1/(1 - x)$ has the power series representation $\sum_{n=0}^{\infty} x^n$ about the basepoint $a = 0$ that converges in $(-1, 1)$. Find a power series representation of $1/(1 - x)$ about the basepoint $a = 2$ (series in powers of $(x - 2)$) and state its radius of convergence. (This example shows that a function can be represented by different power series about different basepoints. After the exam we will use this fact to approximate functions by a polynomial near a basepoint.)

2. Note: Suppose two series about the same point $\sum c_n(x - a)^n$ and $\sum d_n(x - a)^n$ both converge absolutely in an interval $I$. Then the sum of the series converges absolutely in $I$ and equals the series of the sum,

$$\sum a_n(x - c)^n + \sum b_n(x - c)^n = \sum (a_n + b_n)(x - c)^n .$$

Use this fact to find a power series about $x = 0$ for

$$f(x) = \frac{3}{x^2 - x - 2} ,$$

by first using partial fractions. Also state the interval of convergence.

3. Find the sum of the following series in closed form.

(a) $\sum_{n=0}^{\infty} x^{n+2}$ 

(b) $\sum_{n=0}^{\infty} nx^{n+1}$

4. §12.9: # 27 (Write a definite integral as a series and approximate it)